Uncertainty Relations and Quantum Information Security

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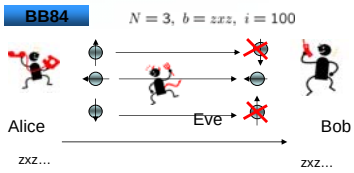
Introduction

The security of Quantum Key Distribution (BB84)

Mayers (1996), Biham-Boyer-Boykin-Mor-Roychowdhury (2000), Shor-Preskill (2000), Koashi (2005), Hayashi (2006)….

Based on Information-Disturbance theorem

An Information theoretical representation of the Uncertainty Relation



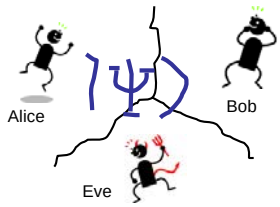
Information gain by Eve induces errors in Bob's outcome = Information-Disturbance theorem

⇩

Alice and Bob can notice the existence of Eve.

Our Generalization [MI4]

BB84= E91



Entropic Uncertainty Relation
Deutsch(1983), Maassen-Uffink(1998), Krishna-Parthasarathy (2002)

$$H(A) + H(B) \geq -2\log\left(\max_{a,b}\|A_a^{1/2}B_b^{1/2}\|\right)$$

$A = \{A_a\}, \quad B = \{B_b\}$ POVMs

A and *B* cannot be *certain* simultaneously, if they are *noncommutative*

noncommutativity

It enables us to treat the general source.

$$I(A:B) + I(\overline{A}:E) \leq H(A) + H(\overline{A}) + 2\log\max_{i,k}\|X[\{p_j,\rho_j\}]_i^{1/2}X[\{q_l,\sigma_l\}]_k^{1/2}\|,$$

Info. Gain by Bob
Ex.) Unbiased case $I(A:B) + I(\overline{A}:E) \leq N$

Info. Gain by Eve

Negative if noncommutative

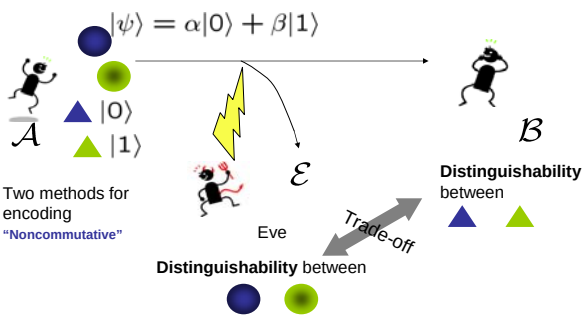
Info. vs. Randomness of Error

[MI1], Hayashi(2006)

$I(A:E|b) \leq H(A \oplus B|\vec{b})$

Another kind of Information-Disturbance

Another measure for dinstinguishability → Another Information-Disturbance theorem

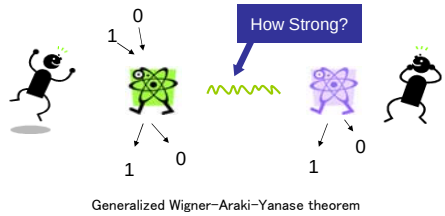


$$\|T^*_{\mathcal{E}}(|\psi\rangle\langle\psi|) - T^*_{\mathcal{E}}(\rho)\|_1 \leq 2|\alpha||\beta|F(T^*_{\mathcal{B}}(|0\rangle\langle 0|), T^*_{\mathcal{B}}(|1\rangle\langle 1|))$$

Measure for distinguishability =Trace distance & fidelity

This formulation can be applied to another quantum impossibility theorem.

The Problem – Information Distribution – [MI2, MI3]



No-cloning theorem for Orthogonal states!

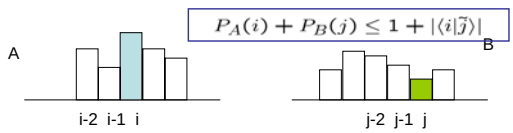
$$|\langle\psi_0|H_S|\psi_1\rangle| \leq \|H_A\|F(\rho_0^S,\rho_1^S) + \|H_S\|F(\rho_0^A,\rho_1^A) + 2\|V_{int}\|$$

If $2\|V_{int}\| < |\langle\psi_0|H_S|\psi_1\rangle|$ holds,

perfect information distribution cannot be attained!

Generalized Landau-Pollak Uncertainty Relation

Landau-Pollak Uncetainty Rel. in its original form Massen-Uffink 1987 (for PVM)



Generalization to Arbitrary num. of General Observables

Theorem [MI5]

Let us consider a Hilbert space $\mathcal{H}$ and a family of m positive operators $\{A_j\}_{j=1}^m$ satisfying $A_j \leq 1$. For any state ρ,

$$\sum_{i=1}^m \langle A_i\rangle_\rho \leq 1 + \left(\sum_{i\neq j}\|A_i^{1/2}A_j^{1/2}\|^2\right)^{1/2},$$

holds.

Summary

- Our generalization enables us to treat the general sources in cryptographic setting.
- Its derivation is based upon the entropic uncertainty relation.
- We can apply the Information-Disturbance theorem for the fidelity and the trace distance directly to the full protocol of BB84.
- It is useful in deriving other quantum impossibilities: Wigner-Araki-Yanase theorem, Heisenberg uncertainty relation.
- We generalize the Landau-Pollak uncertainty relation.

[MI1] T. Miyadera and H. Imai, Information-Disturbance theorem for Unbiased Observables, Phys.Rev.A 73 042317 (2006)
[MI2] T. Miyadera and H. Imai, Wigner-Araki-Yanase theorem on Distinguishability, Phys.Rev.A 74 024101 (2006)
[MI3] T. Miyadera and H. Imai, Strength of Interaction for Information Distribution, Phys.Rev.A **74** 064302 (2006)
[MI4] T. Miyadera and H. Imai, Information-Disturbance theorem and Uncertainty Relation, arxiv.0707.4559
[MI5] T. Miyadera and H. Imai, Generalized Landau-Pollak Uncertainty Relation, Phys.Rev.A **76**, 062108 (2007)