Code-Based PKCs And Their Applications

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PKCs can be divided into

Number Theoretic (Cyclic) Problem

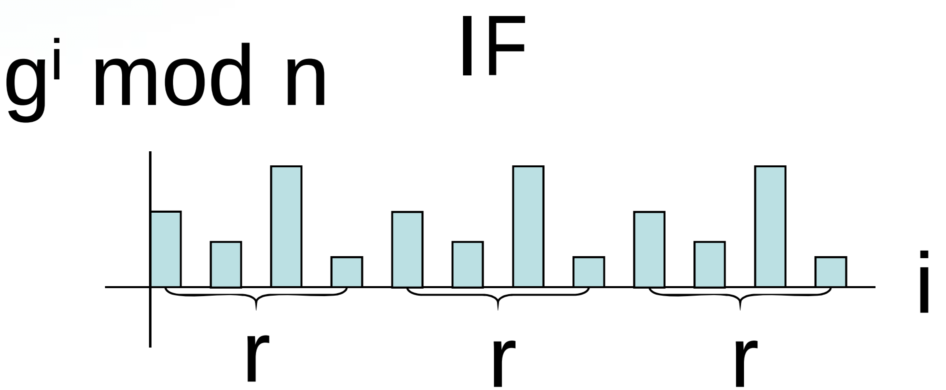
- Integer Factoring Based
  - RSA[RSA78]
  - Rabin[Ra79]
  - Okamoto-Uchiyama[OU98]
  - Paillier[Pa99]
  - S-Paillier[CHGN01]
  - …
- Discrete Logarithm Based
  - Diffie-Hellman[DH77]
  - ElGamal[El84]
  - ECC[Mi85][Ko87]
  - XTR[LV00]
  - Cramer-Shoup[CS03]
  - Kurosawa-Desmedt[KD04]
  - …

Combinatorial Problem

- Code Based
  - McEliece[Mc78]
  - Niederreiter[Ni86]
- Lattice Based
  - NTRU[HS96]
  - AjtaiDwork[AD97]
  - Goldreich-Goldwasser-Halevi [GGH97]
  - Ajtai[Ajt05]
  - Regev[Reg03,Reg05]
  - Peikert[Pei09]
  - …
- Subset Sum Based
  - Okamoto-Tanaka-Uchiyama[OTU00]

Cyclic Problem: Integer Factoring (IF)

- Given a positive integer n, find its prime factor p_i
- Equivalent to finding r s.t. $g^r \equiv 1 \pmod n$
  - for a g of $GCD(g,n)=1$ and $g \not\equiv \pm 1 \pmod n$
- Since $GCD(g^{r/2}-1,n)$ or $GCD(g^{r/2}+1,n)$ is p_i



This cycle can be determined in poly time with 1D-QFT, a.k.a. Shor's quantum algorithm.

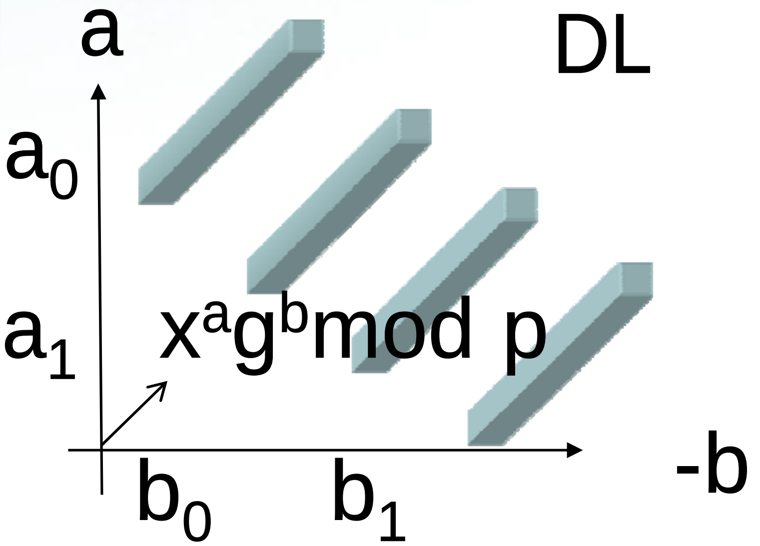
QFT : Quantum Fourier Transform

Cyclic Problem: Discrete-Log (DL)

- Given p, g and y, find r s.t. $x \equiv g^r \bmod p$
  - p: prime, g: generator of Z_p*, x a member of Z_p* except 1, p-1.
- Equivalent to finding a_0, a_1, b_0 and b_1 s.t.

$$x^{a_0} g^{b_0} \equiv x^{a_1} g^{b_1} \bmod p$$

  - Since $x^{(a_1-a_0)} \equiv g^{r(a_1-a_0)} \equiv g^{(b_0-b_1)} \bmod \mathrm{p}$



$$r \equiv \frac{(b_0 - b_1)}{(a_1 - a_0)} \bmod \mathrm{p}-1$$

This inclination can be determined in poly time with 2D-QFT, a.k.a. Shor's quantum algorithm.

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Lesson to learn

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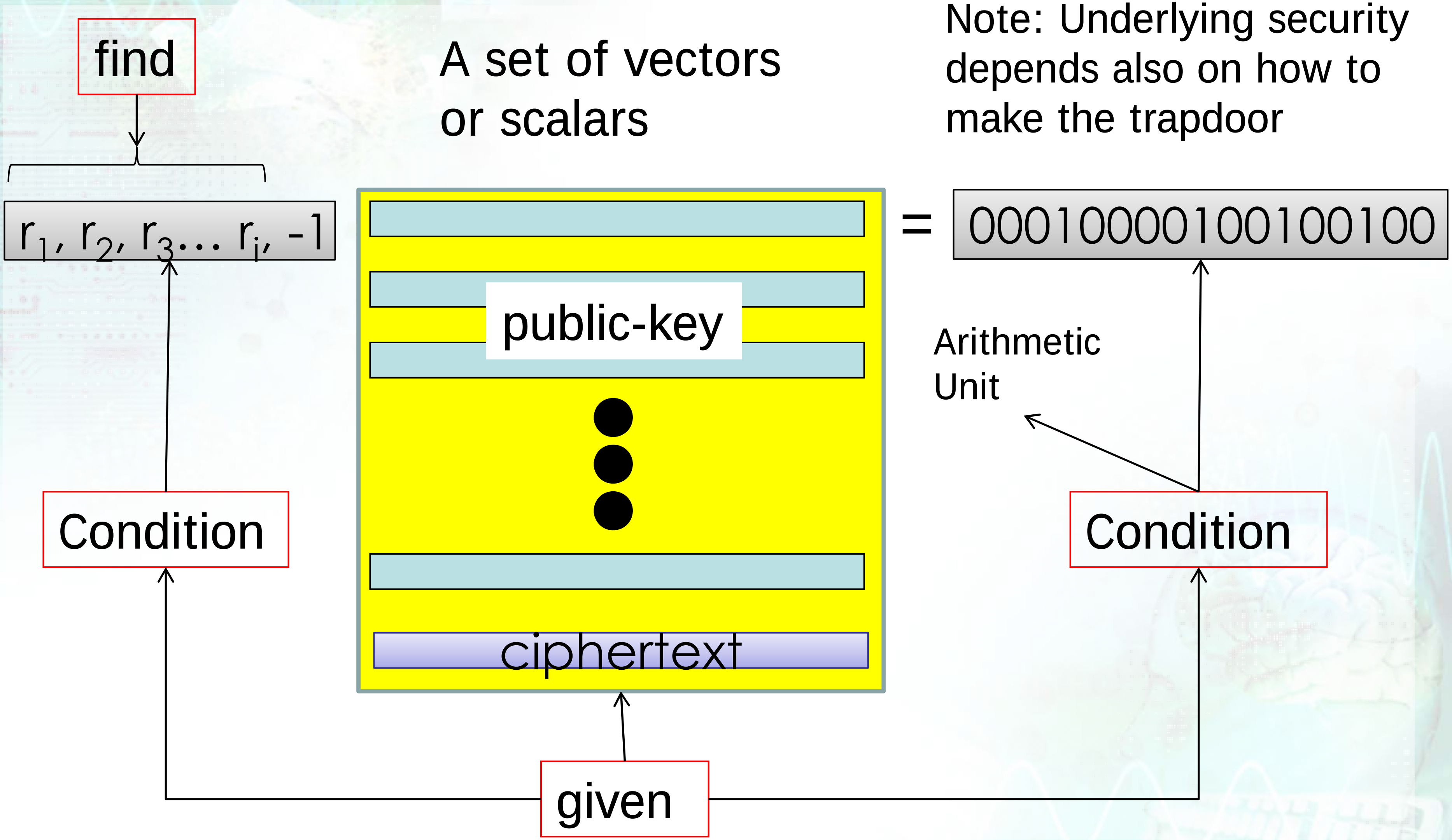
- If a problem is written in the form of determining a cycle,
- Then it can be solved in polynomial time using a quantum computer.

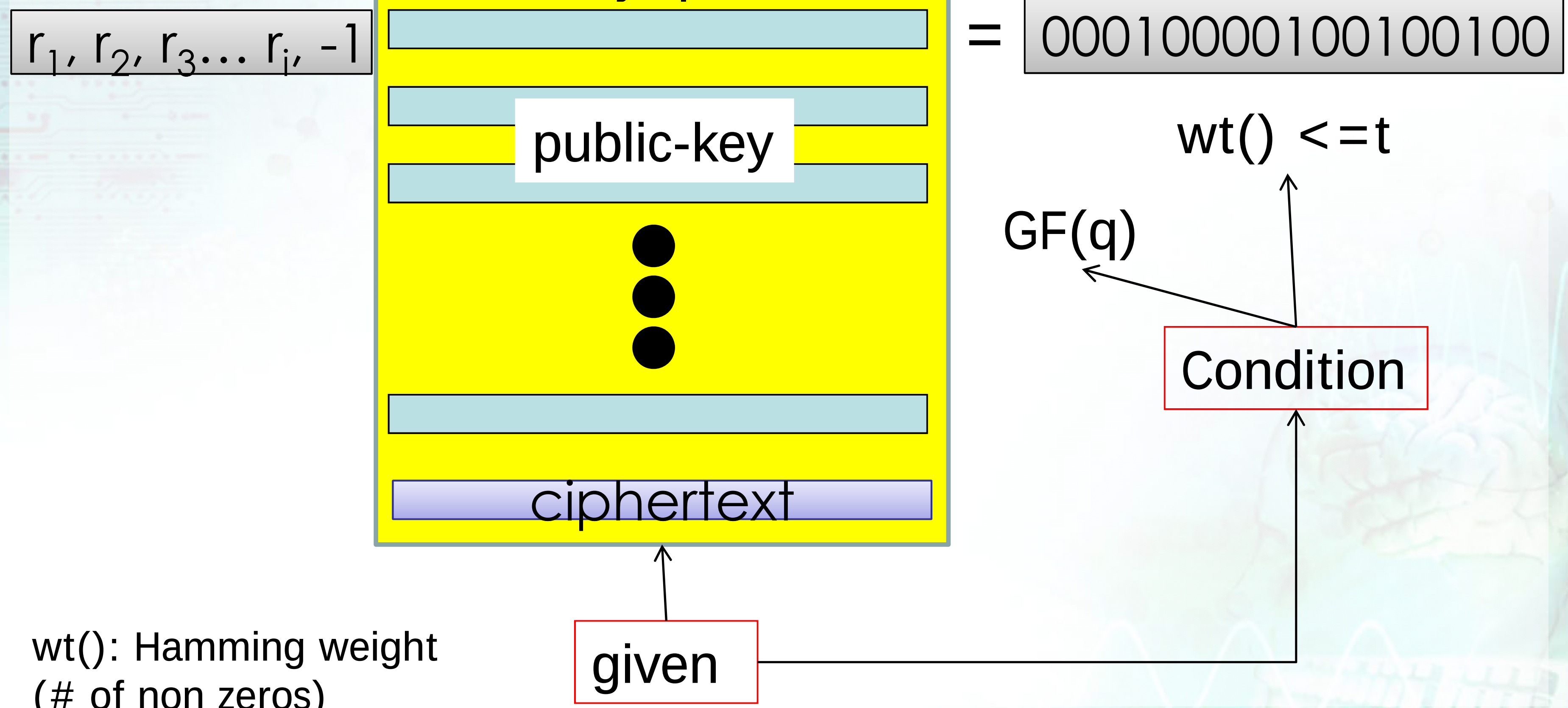
Combinatorial Problems

Note: Underlying security depends also on how to make the trapdoor



Code Based: McEliece

PK is a kxn generator matrix of a linear code over GF(q) , usually q=2



wt(): Hamming weight
(# of non zeros)

Code Based: Niederreiter

PK is a nx(n-k) parity check matrix of a linear code over GF(q), usually q=2

find

Plaintext

$r_1, r_2, r_3 \ldots r_i, -1$

wt(r) <=t

$r \in \{r_1, r_2, \cdots, r_i\}$

public-key

ciphertext

= 0000000000000000000

All Zero

GF(q)

Condition

Condition

given

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Lattice Based: NTRU

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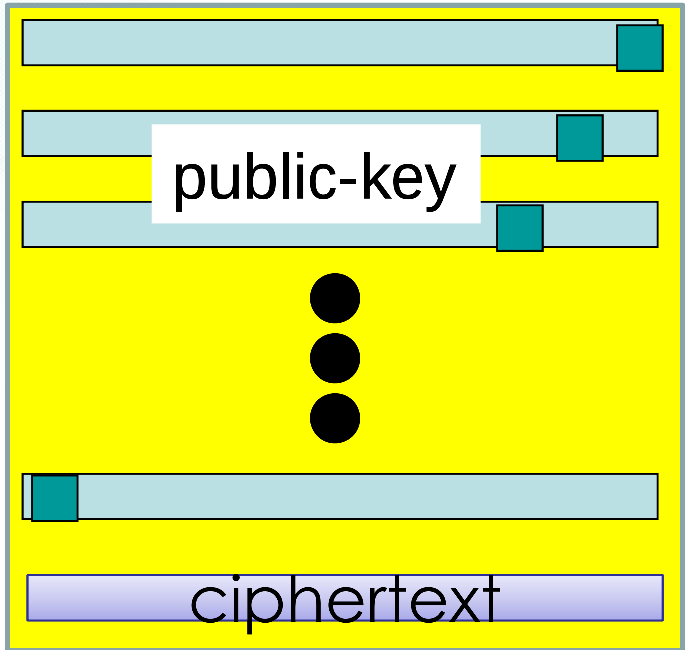
find

PK is a n dimensional cyclic vector over Z_q , q=2048 etc.

Plaintext

$r_1, r_2, r_3 \dots r_i, -1$



= 00100-100100-10000

$r_i \in \{-1,0,1\}$,

$\mathrm{wt}() = \mathrm{d}_\mathrm{r}$

$m_i \in \{-1,0,1\}$, $\mathrm{wt}() = \mathrm{d}_\mathrm{m}$

Z_q

Condition

Condition

given

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Lattice Based: Regev

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PK is a (u+v)x(n+v) matrix

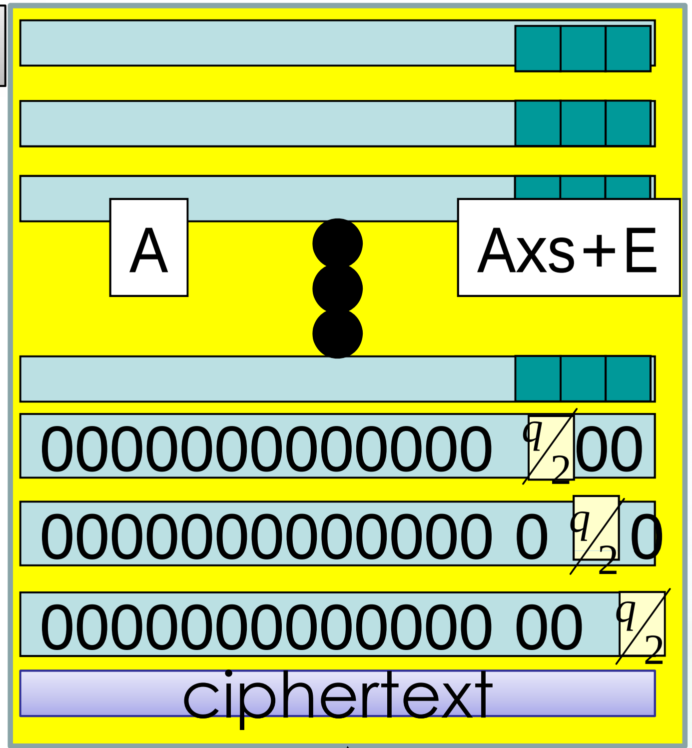
uxn random binary matrix A

uxv parity-with-noise Axs+E

find

Plaintext

$r_1, r_2, \ldots, r_i, m_1, \ldots, m_v, -1$



= 0000000000

All Zero

Z_q

$$r_i, m_i \in \{0,1\},\ \mathrm{wt(r)} < \frac{q}{2}$$

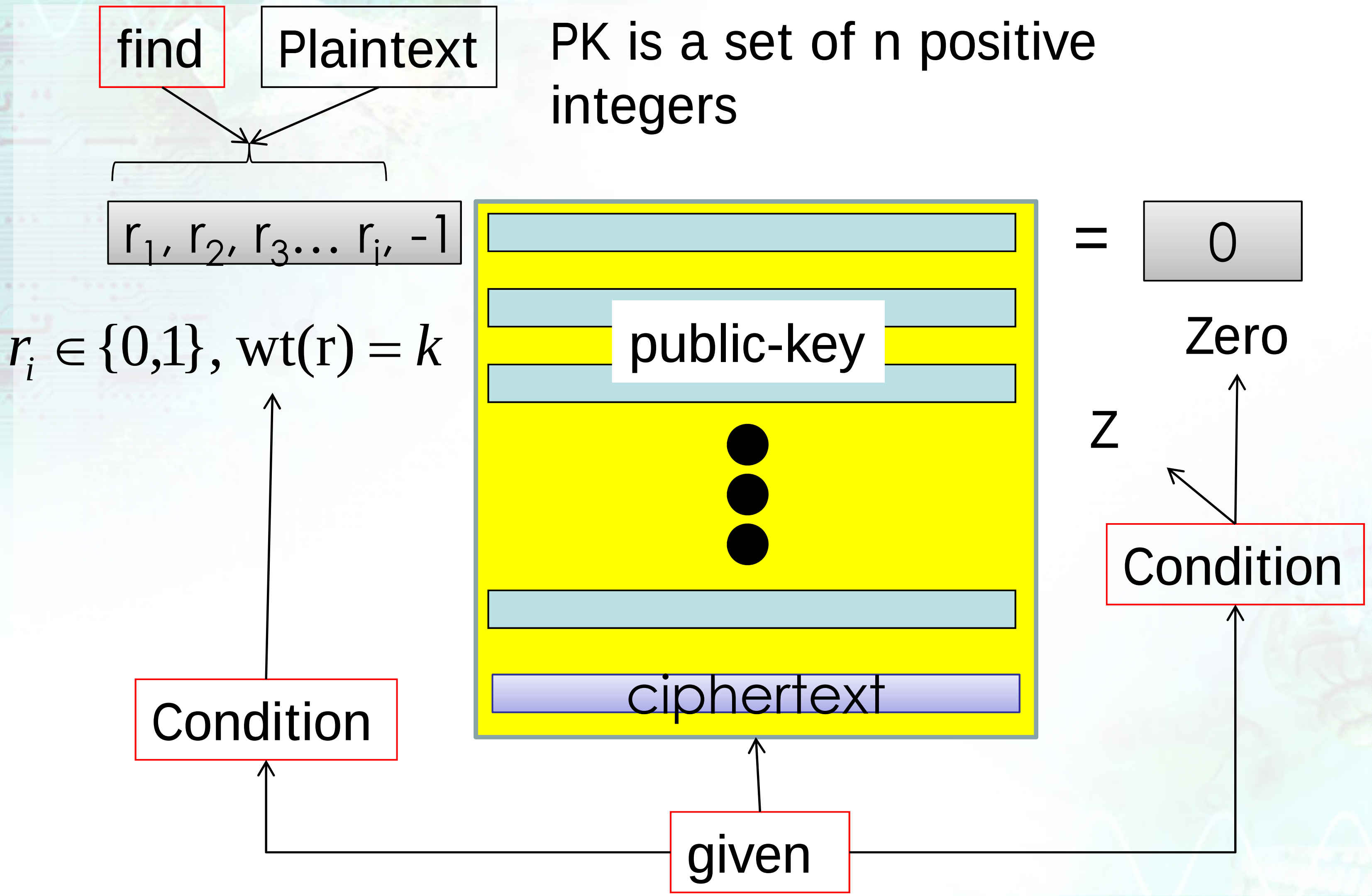
Condition

Condition

given

Subset Sum Problem: OTU

PK is a set of n positive integers



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So far it is not known

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- To reduce combinatorial problems to a cyclic problem
- The best attack on combinatorial problem is:
  - Grover's algorithm
- Grover's algorithm can reduce
  - Running time to $\sqrt{\text{states}}$ but it is still exponential to its input size

Advantages of Combinatorial Based

- Quantum Tolerant:
  - No polynomial time algorithm is known so far even on quantum computers
- Arithmetic unit is small:
  - for encryption (and signature verification)
  - Usually xors or additions in a small Field/Ring
  - They are highly parallelizable
  - No heavy multi-precision modular exponentiation

Advantages of Code-Based

- Information Ratio, i.e.

$$\frac{(\text{Plaintest Size})}{(\text{Ciphertext Size})}$$ is better

- Arithmetic unit is smaller:
  - for encryption and signature verification
  - Usually xors

=> Suitable for low computational power or Ubiquitous devices

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History of Code-Based PKEs

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Code-based

1978: McEliece

1986: Niederreiter

2000: Error expansion/hold [Loi00]

Attack [KI03]

Attack on GRS [SS92]

Attack on small cardinality including BCH [LS01]

Goppa code (more generally alternant code) with large cardinality

2001: IND-CCA2 Conv. in RO [KI02]

2005: Quasi-Cyclic BCH subcode [Gab05]

LDPC

2007: QC-LDPC [BC07]

2005: with GRS subcode [BL05]

Attack on GRS subcode [Wie06] [Wie09]

2009: Quasi-Dyadic [MB09]

2009: Quasi-Cyclic alternant code [BCG+09]

Attack on QC [OTD08]

Large extension degree

Attack on small extension degrees [UL09]

A lot of attacks where some of them were critical like reaction attacks

Number Theoretic

1977 Hiffie-Hellman

1977: RSA

1985: ECC

1991: GPT PKC [GPT91] (rank code)

Attack on sparse S [MRS00]

Attack on LDPC in general

Attack on subfield P [Ove05]

2008: ext field P [Gab08]

Idea can be applied to

How to see the history

- Green Letters: Underlying Liner Code
- ← : Attack
- ✕ : Already Broken
- ◎ : Not Yet Broken

(as far as I know as of Dec. 2009)

History of Non PKE Code-Based Primitives

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ZKIP: Zero Knowledge Interactive Proof
OT: Oblivious Transfer

1994: ZKIP (as well as Digital Signature applying Fiat-Shamir heuristic) (random code) [Ste94]

Public matrix can be compressed by generating it from a random seed

2001: Digital Signature (Goppa) [CFS01]

Bleichenbacher's attack [Ble06]

2005: Hash Function (random code) [AFS05]

2007: Light Weight Unlinkable-ID for RFID (Goppa) [SKI06][CKM+07]

2008: Bit (1,2)-OT (Goppa) [DGQ+08]

2008: String (1,2)-OT (Goppa) [KMO08]

With appropriate parameter choice [FS09]

Construction of Code-Based PKCs

Keys for McEliece PKE

- Secret key

Random Permutation Matrix

Non-Singular Matrix

S $\quad$ $k\Big\{\overbrace{G}^{n}$ $\quad$ P

Generator matrix of ECC
which can correct up to t-error symbols

- Public key

$$\| $$

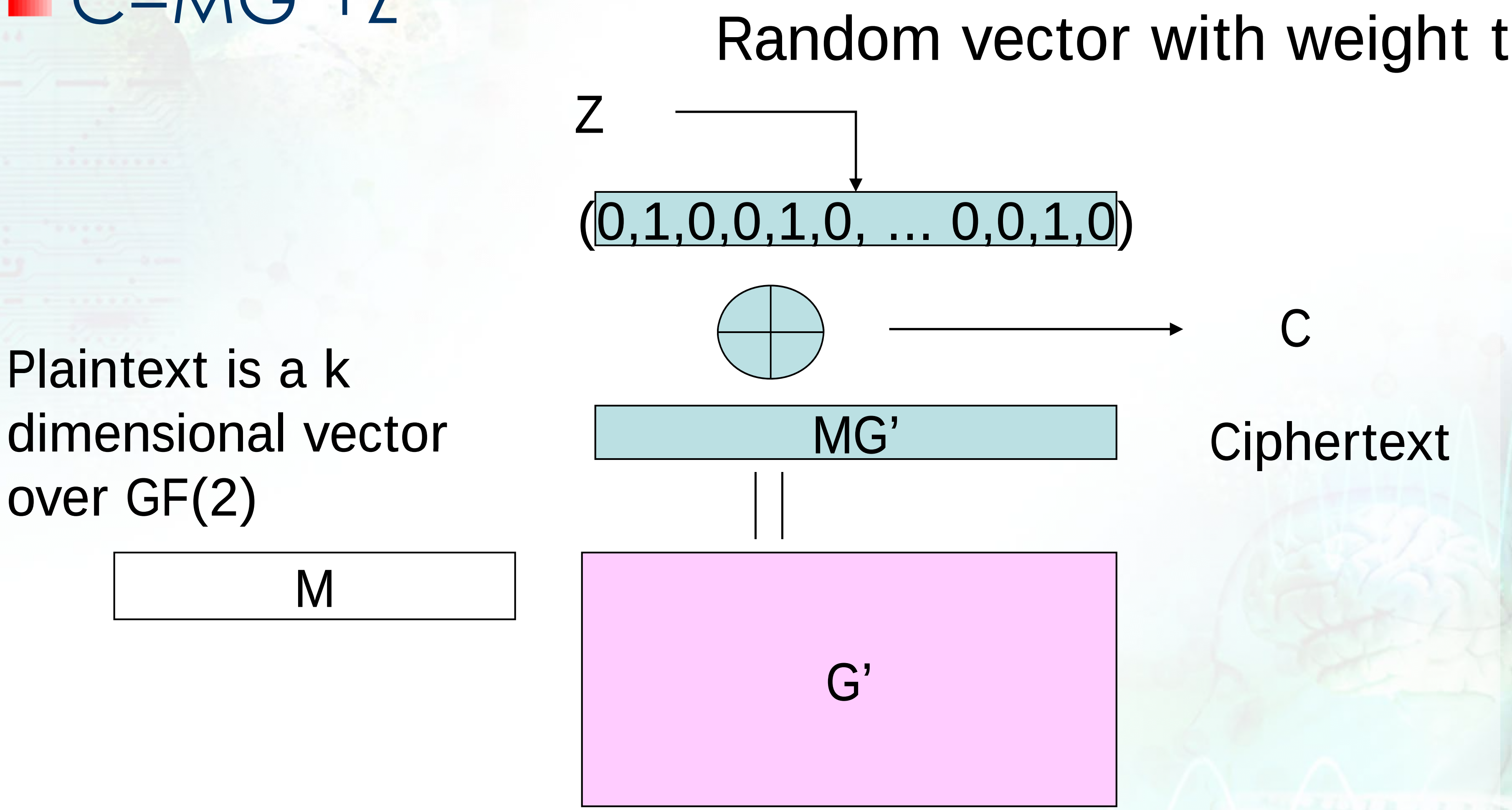
G'

May be systematic if an appropriate IND-CCA2 conversion is applied

$[\,I \mid G''\,]$

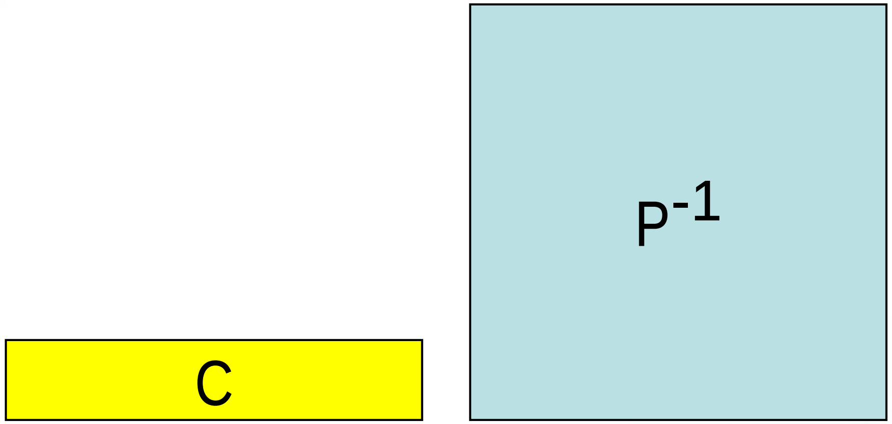
Encryption of Primitive McEliece PKE

- C=MG'+Z

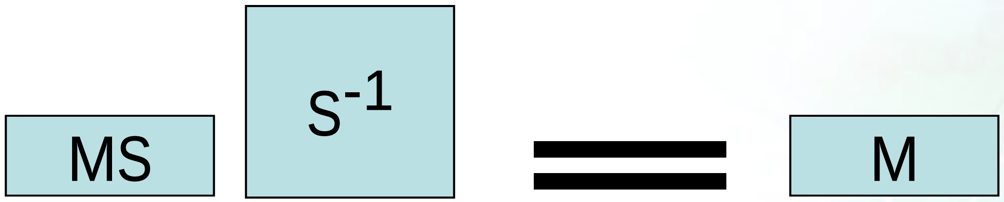


Decryption of Primitive McEliece PKE

- 1. $CP^{-1}=MSG+ZP^{-1}$



- 2. Correct the t-error bits using error-correction algorithm and obtain MS
- 3. $M=MS\ S^{-1}$



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Keys for Niederreiter PKE

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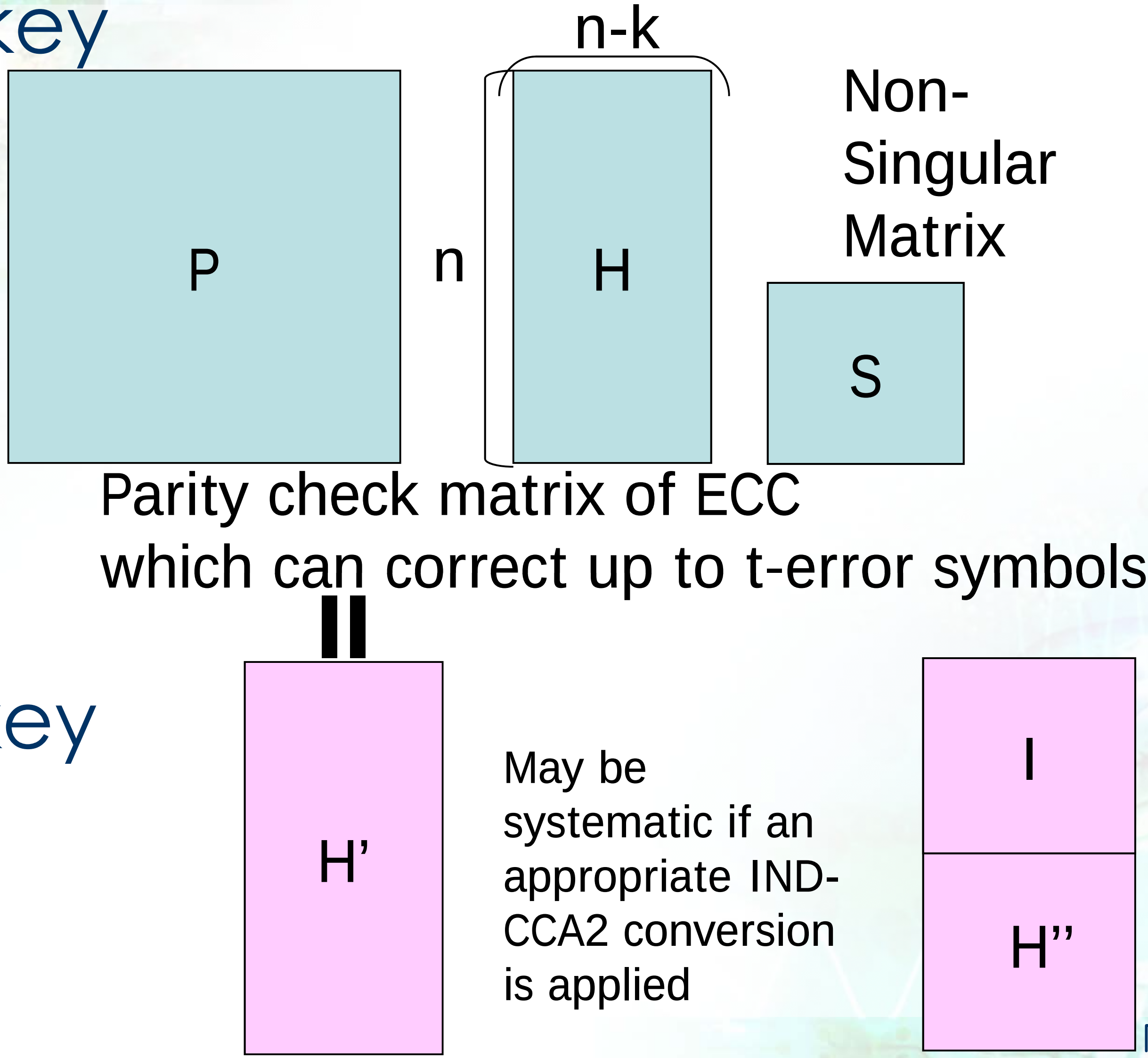
- Secret key

Random Permutation Matrix

Non-Singular Matrix



- Public key

Encryption of Primitive Niederreiter PKE

- C=ZH'

Plainstext is an n dimensional vector of weight t

Ciphertext

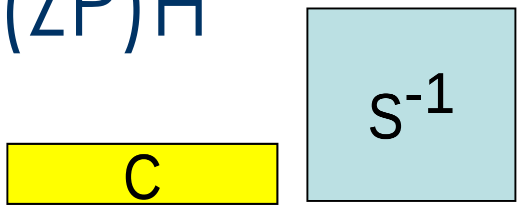
Z

(0,1,0,0,1,0, … 0,0,1,0) H' = ZH'

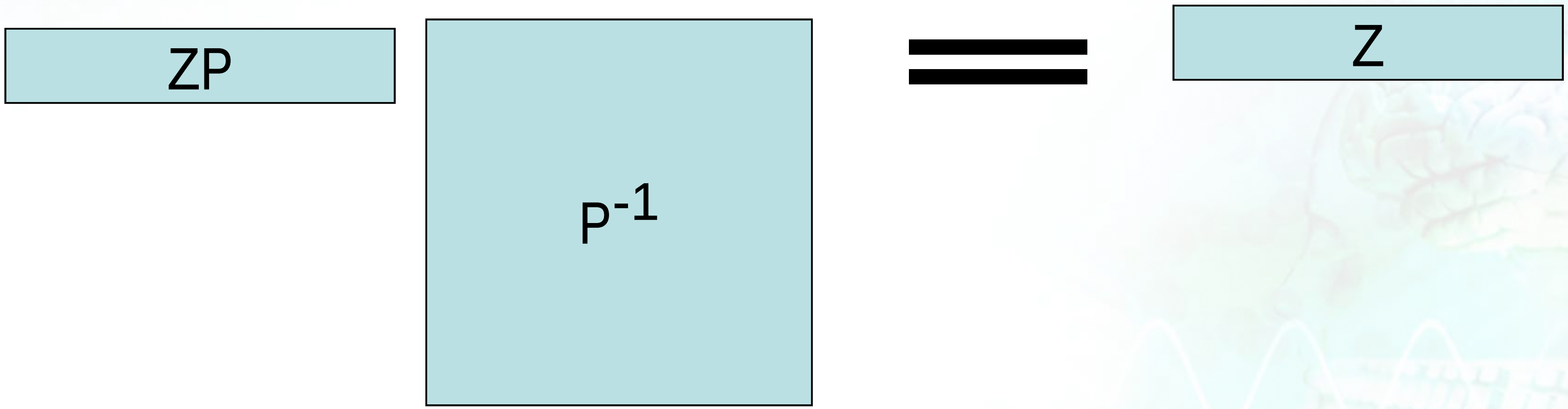
C

Decryption of Primitive Niederreiter PKE

- 1. $CS^{-1}=ZH'S^{-1}=ZPH\ SS^{-1}=(ZP)H$



- 2. Correct the t-error bits using error-correction algorithm and obtain ZP
- 3. $Z=ZP\ P^{-1}$



Unfortunately

- Due to the simple structure of a linear code, a lot of practical attacks are known
  - This is the reason why some people still think that code-based PKEs have already been broken
- Ex.)
  - Reaction attack can decrypt a given ciphertext in O(k).

Reaction Attack on McEliece PKC

- Flip one bit of the given ciphertext
- Let the receiver decrypt it
- If its reaction is normal, error is within the error correction bound.

1

$\oplus$

C= 0 1 1 0 0 0 1 0 1 0 1 1 0 1

Given ciphertext

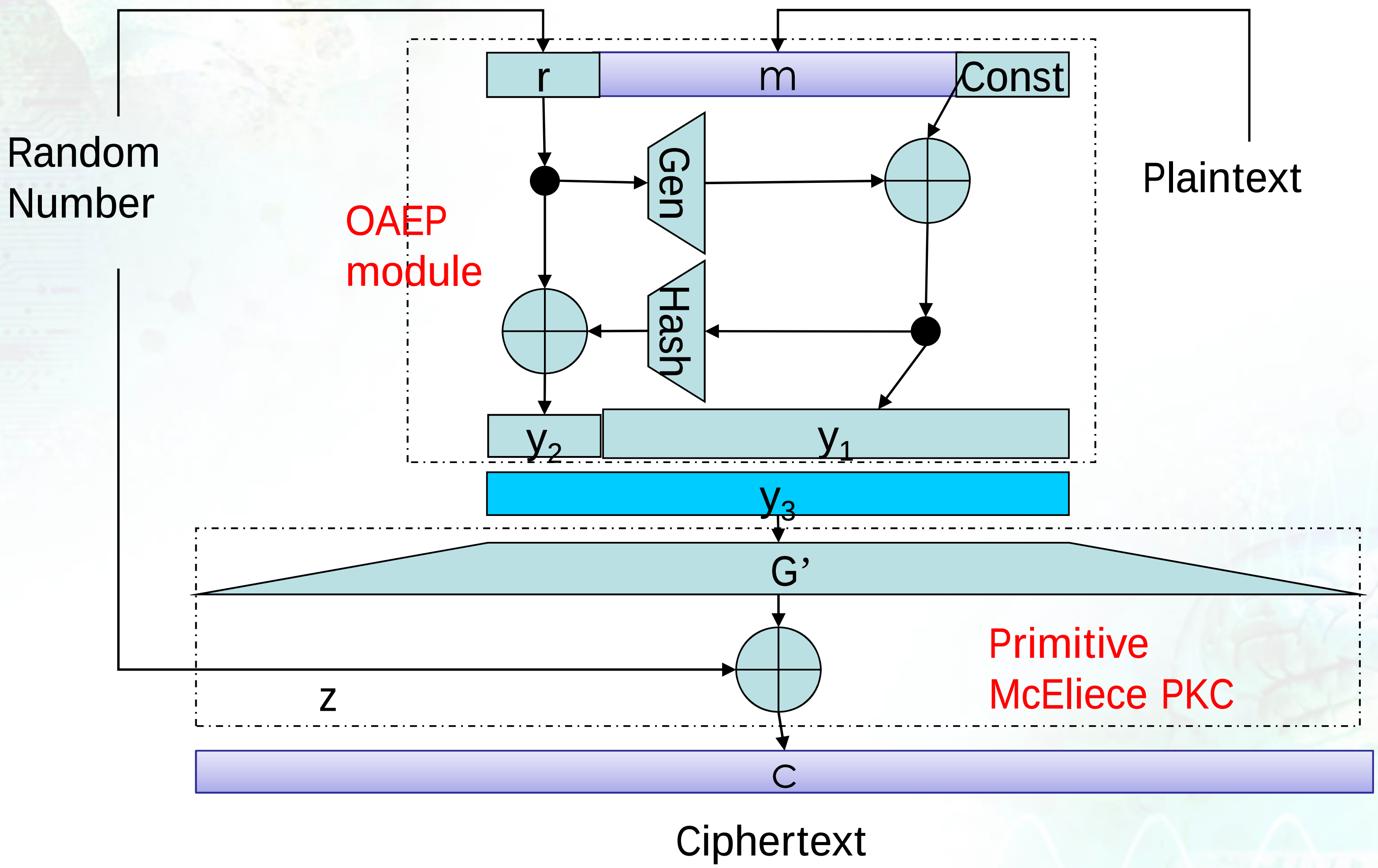
Receiver (decryption oracle)

Reaction

Countermeasures

- Attacks
  - Using decryption oracles
  - Against indistinguishability or non-malleability
- Can be prevented applying an "appropriate" conversion scheme
  - Naïve application sometimes does not work

Naïve application of OAEP is vulnerable



Slight Modification Makes it Provably Secure and Compact

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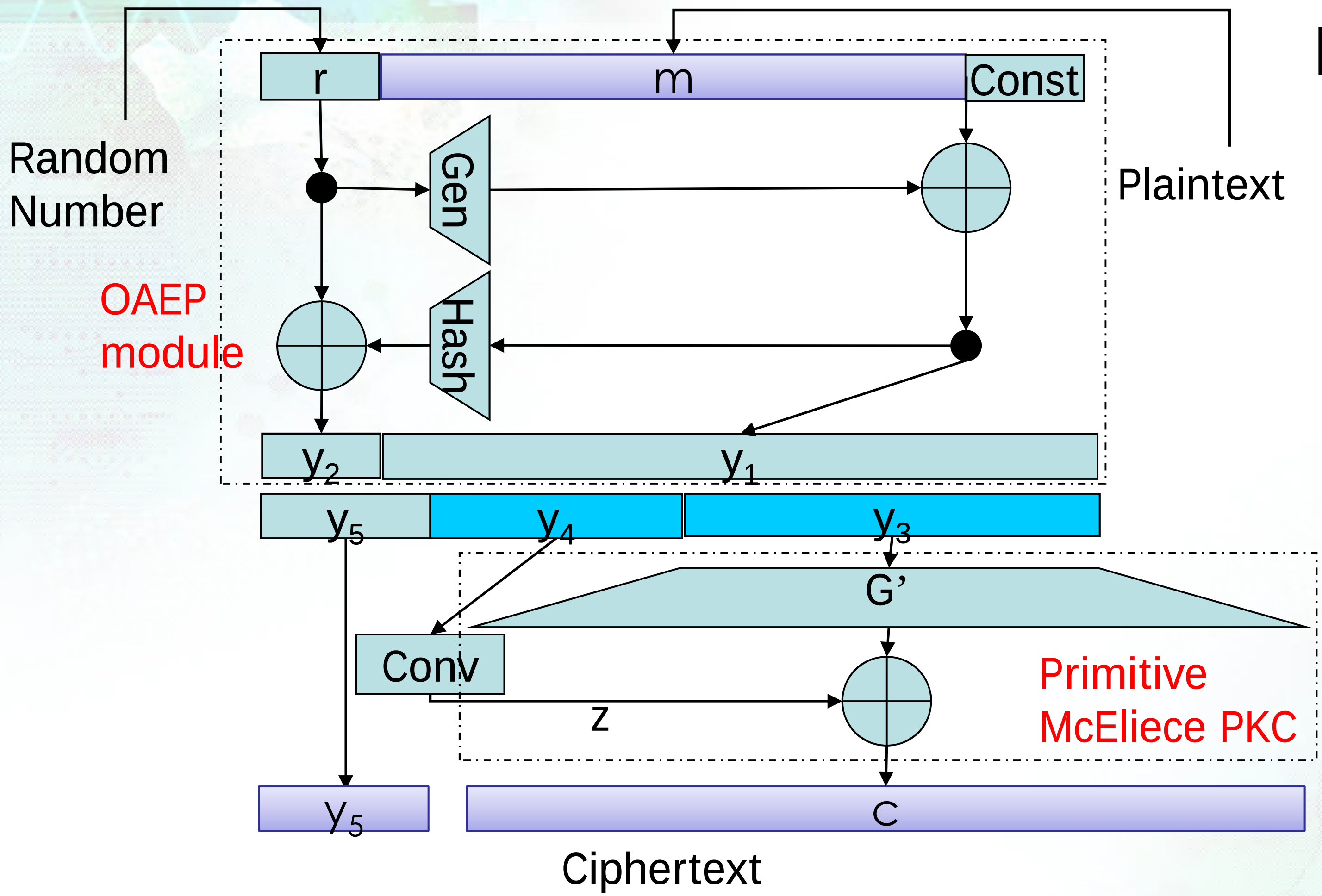
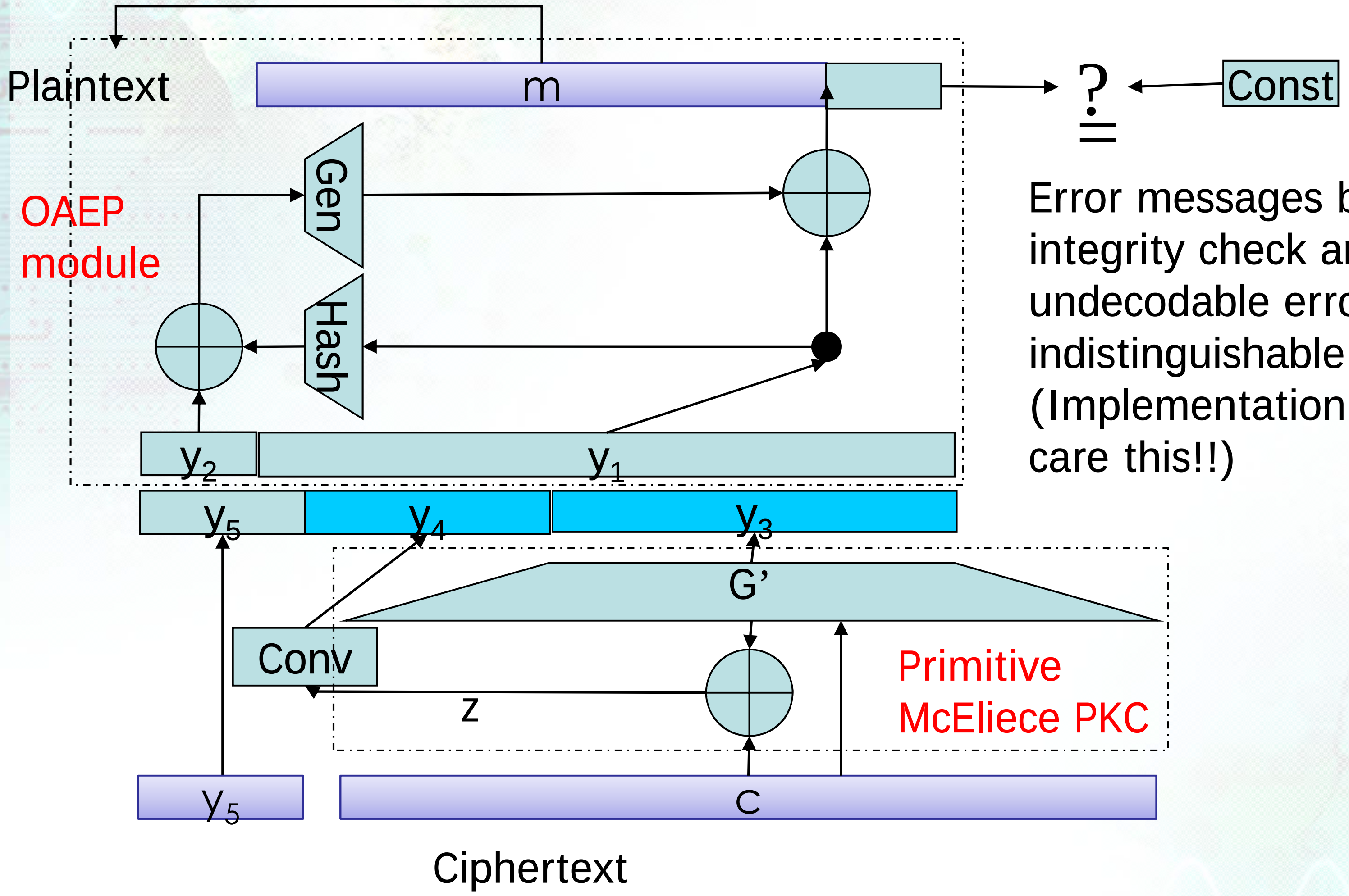
Encryption



$(Len(y_2)+Len(y_1)\geq Len(y_4)+Len(y_3))$ and must hold

$((Len(y_1)\geq Len(y_3))$ or $(Len(y_1)\geq Len(y_4)$ by swapping y_3 and $y_4))$

Decryption of Conversion



Error messages between this integrity check and undecodable error must be indistinguishable. (Implementation must take care this!!)

In The Case of Niederreiter

- Make the ciphertext most compact and provably secure in the RO
  - OAEP+ for short plaintext
  - OAEP++ for long plaintext
- No proof but no attack
  - OAEP
  - SAEP+
- Insecure (since Niederreiter primitive is position-wise malleable)
  - SAEP

Provable Security in the Standard Model

- 2007: IND-CPA in the standard model [NIK+07]
- 2008: IND-CCA2 in the standard model [DQN08]

IND: Indistinguishability
CPA: Chosen Plaintext Attack
CCA2: Adaptive Chosen Ciphertext Attack

Secure Constructions are Available

- As long as the primitive McEliece/Niederreiter PKC satisfies OW-CPA
- Parameters meeting OW-CPA against most powerful attacks ISD and GBA are estimated in [FS09]

OW-CPA: One-Wayness against Chosen Plaintext Attack
ISD: Information Set Decoding
GBA: Generalized Birthday Attack

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Parameters for PKE

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m	t	Binary work factor	PK size (KB)	Plaintext / Ciphertext size	
				McEliece	Niederreiter
11	32	$2^{86.8}$	72.9KB	1696/2048 bits	233/352bits
12	41	$2^{128.5}$	216.5KB	3604/4096 bits	327/492bits

Public-key size is large compared to Number Theoretic ones.

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Parameters for Digital Signature

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m	t	Binary work factor	PK size (KB)	Iteration *1	Signature Size *2,*3
22	9	$2^{81.7}$	101,371KB	$2^{18.5}$	198～216.5 bits
15	12	$2^{81.5}$	716.0KB	$2^{28.8}$	180～208.8 bits
14	13	$2^{80.7}$	360.0KB	$2^{32.5}$	182～214.5 bits
13	14	$2^{80.0}$	178.0KB	$2^{36.4}$	182～218.4 bits
12	15	$2^{88.2}$	86.0KB	$2^{40.3}$	180～220.3 bits

*1: affects the signing cost

*2: Signature size depends on how to express the error pattern and it affects the verification speed.

*3: Signature size can be reduced further by removing some error positions while increasing the verification cost further [CFS01]

How to Reduce Public-Key Size

- Increase the error correction capability
  - Capacity Approaching Codes
    - LDPC, QC-LDPC ✗
  - List Decoding △
    - corrects only a couple of more errors for practical parameters while increasing the decoding complexity
- Compress the public-key
  - Quasi-Cyclic ✗
  - Quasi-Dyadic
    - Small extension degree ✗
    - Large extension degree ◎

Attack on LDPC Codes

Exploits the fact the density of the secret matrix low.

H'^T = S | H^T: Parity Check Matrix of LDPC code | P

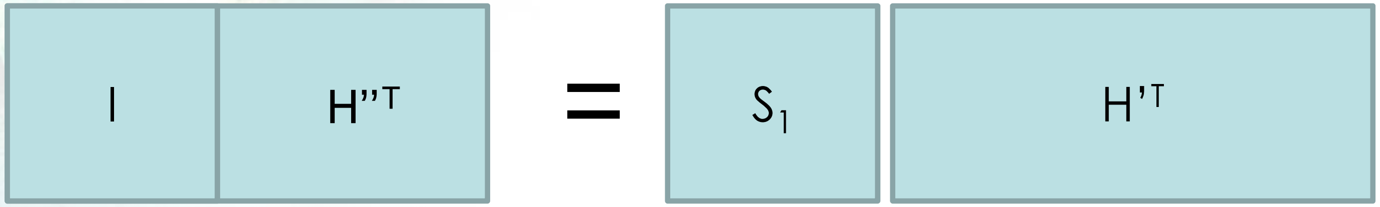
This may be hidden with S and P but can be recovered.

Let

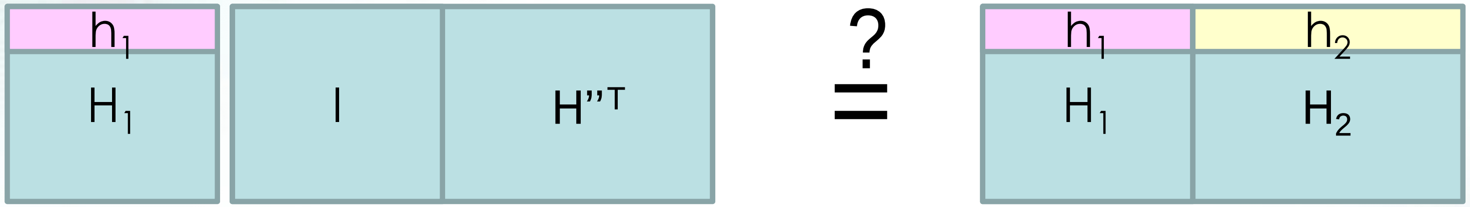
H^T: Parity Check Matrix of LDPC code = H_1 | H_2

Attack on LDPC Codes

1. Make H' systematic



2. Multiply a low density vector h_1 , and check if the density of h_2 is low.

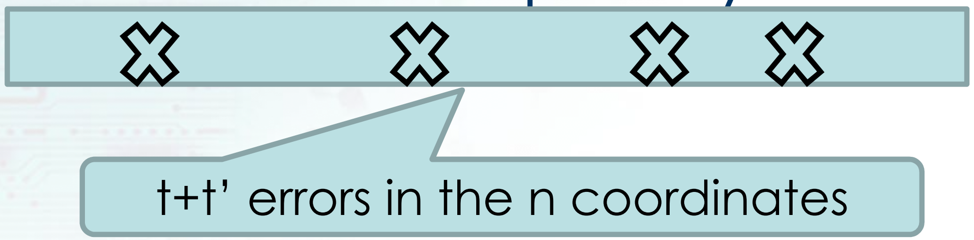


Since # of candidates C(n-k,w) where w is the Hamming weight of h_1 (and also the possibility of being another low density matrix is negligible), H can be recovered.

If one uses this direction, they must find a good code of Middle Density (we call MDPC).

List Decoding

- Exhaustive Search
  - Correct any t' more errors but with the complexity $O\left(\frac{\binom{n}{t'}}{\binom{t+t'}{t'}}\right)$



$$\left(\frac{n-t'+1}{t+1}\right)^{t'} \geq \frac{\binom{n}{t'}}{\binom{t+t'}{t'}} \geq \left(\frac{n}{t+t'}\right)^{t'}$$

- Bernstein's List Decoding [Ber08]
  - t' more errors in poly time where

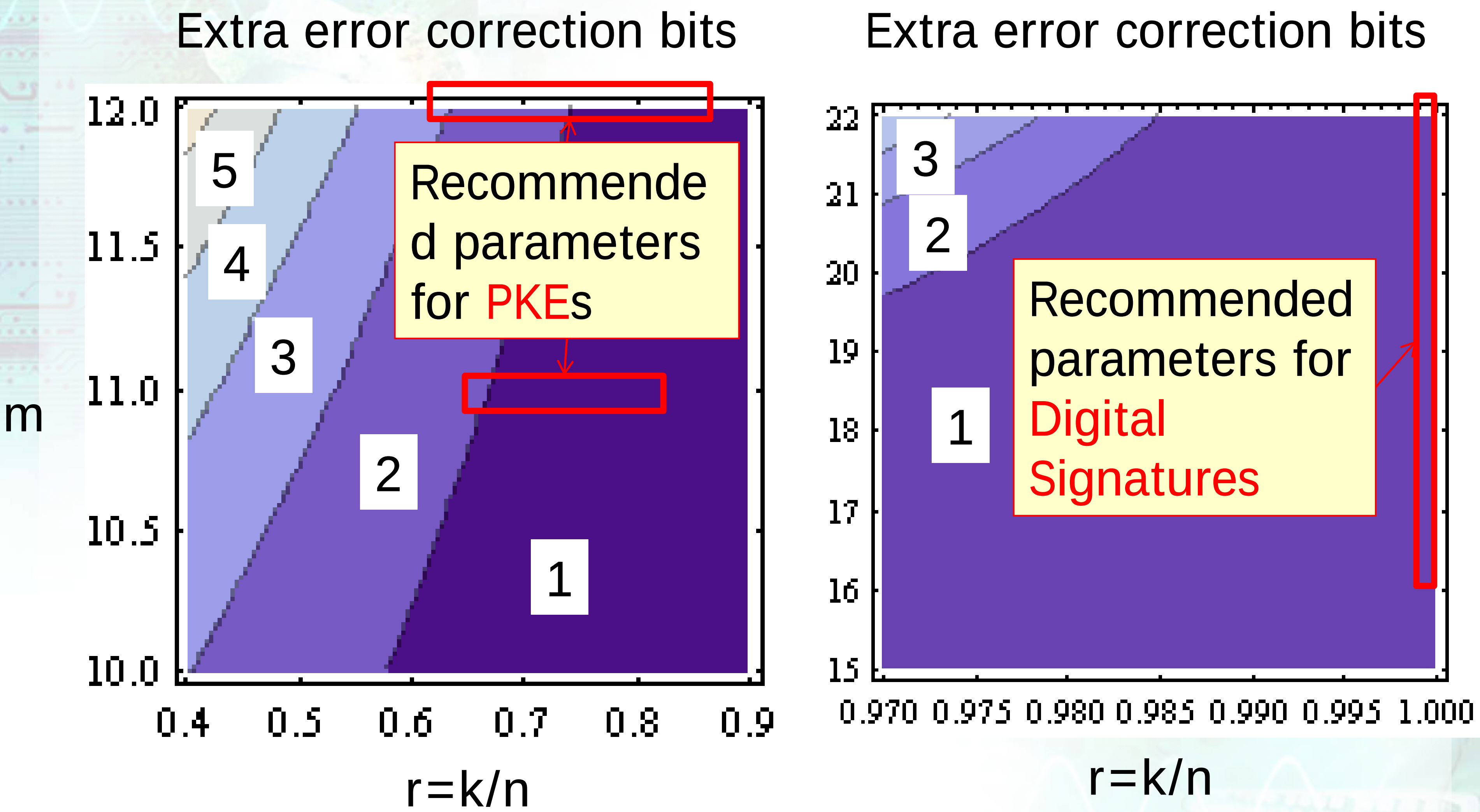
$$t' = n - \sqrt{n(n-2t-2)} - t$$

$$= 2^m - \frac{2^m(1-r)}{m} - \sqrt{2^m\left(2^m - 2\left(\frac{2^m(1-r)}{m}\right) - 2\right)}$$

$$n = 2^m$$

$$t = \frac{(n-k)}{m} = \frac{2^m(1-r)}{m}$$

of extra error-correctable bits



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Dyadic Matrix

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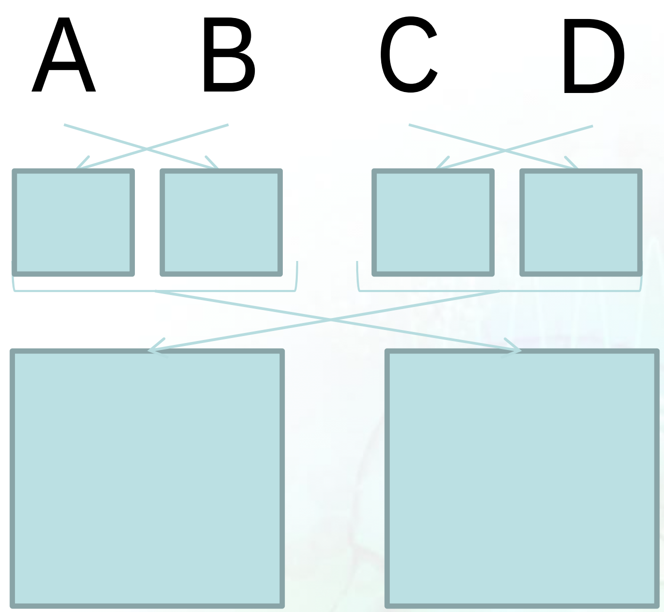
$$\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} A & B \\ B & A \end{bmatrix} & \begin{bmatrix} C & D \\ D & C \end{bmatrix} \\ \begin{bmatrix} C & D \\ D & C \end{bmatrix} & \begin{bmatrix} A & B \\ B & A \end{bmatrix} \end{bmatrix}$$

Advantage of Dyadic Matrix

$$\begin{bmatrix} \begin{bmatrix} A & B \\ B & A \end{bmatrix} & \begin{bmatrix} C & D \\ D & C \end{bmatrix} \\ \begin{bmatrix} C & D \\ D & C \end{bmatrix} & \begin{bmatrix} A & B \\ B & A \end{bmatrix} \end{bmatrix}$$



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Dyadic Matrix and Goppa Codes

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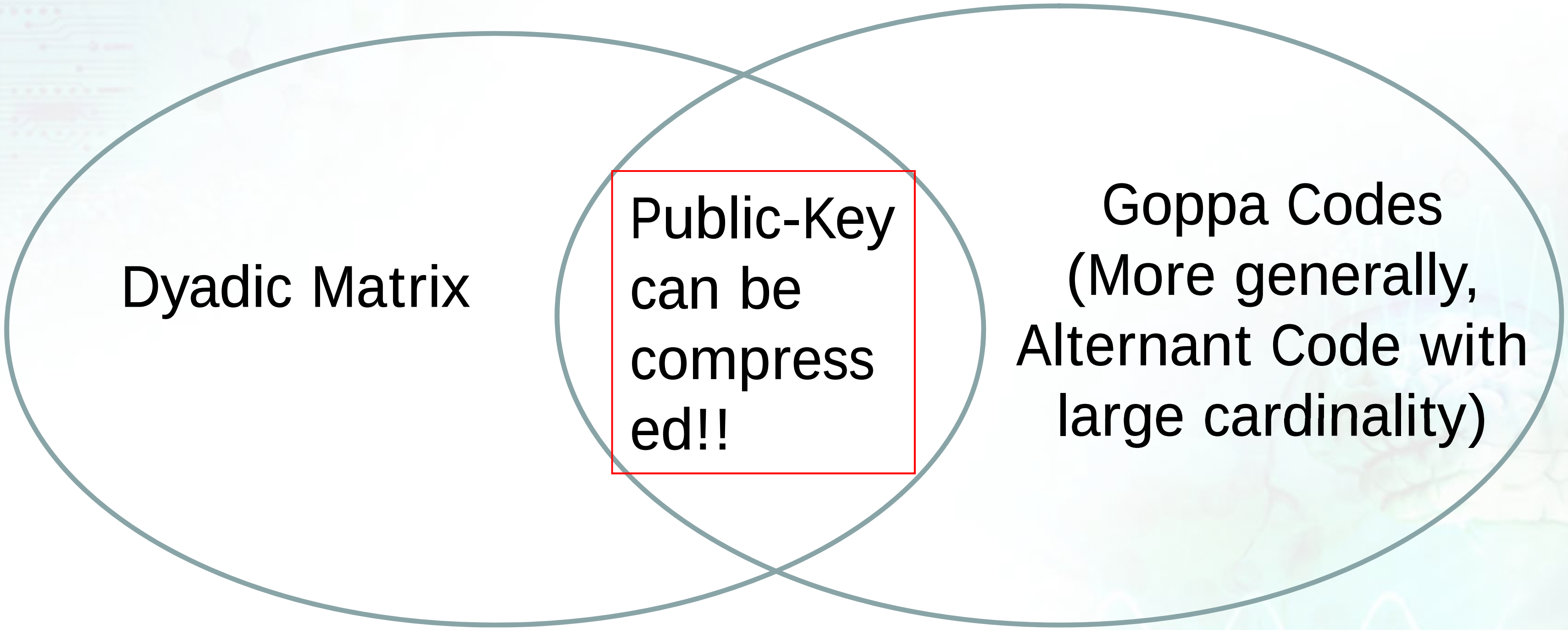
have an intersection over the extension field of GF(2)



Goppa Codes and Cauchy Matrix

- Binary Goppa code is the set of all

$$c=\begin{pmatrix} c_0 & c_1 & \cdots & c_{n-1}\end{pmatrix}\in GF(2)^n$$ s.t.

$$S_e(X)\equiv\sum_{i=0}^{n-1}\frac{c_i}{X-L_i}\equiv 0 \bmod g(X)$$

If $g(X)=\prod_{i=0}^{t-1}(z_i-x)$ where z_i and L_j are distinct it can be written

$$(cH)^T=\begin{bmatrix} \frac{1}{z_0-L_0} & \frac{1}{z_0-L_1} & \cdots & \frac{1}{z_0-L_{n-1}} \\ \frac{1}{z_1-L_0} & \frac{1}{z_1-L_1} & \cdots & \frac{1}{z_1-L_{n-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{z_{t-1}-L_0} & \frac{1}{z_{t-1}-L_1} & \cdots & \frac{1}{z_{t-1}-L_{n-1}} \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{n-1}\end{bmatrix}=0$$

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To Make It Dyadic
(more generic than [MB09])

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$\oplus\delta$ can be applied to either of a numerator, a denominator or a fraction

$\oplus\delta_3$

$\oplus\delta_2$ $\oplus\delta_2$

$\oplus\delta_1$ $\oplus\delta_1$ $\oplus\delta_1$ $\oplus\delta_1$

$$
\begin{array}{c c}
\begin{array}{c} \oplus\delta_1 \\ \oplus\delta_2 \\ \oplus\delta_1 \\ \oplus\delta_3 \\ \oplus\delta_1 \\ \oplus\delta_2 \\ \oplus\delta_1 \end{array} &
\left[\begin{array}{cc|cc|cc|cc}
\frac{1}{z_0-L_0} & \frac{1}{z_0-L_1} & \frac{1}{z_0-L_2} & \frac{1}{z_0-L_3} & \frac{1}{z_0-L_4} & \frac{1}{z_0-L_5} & \frac{1}{z_0-L_6} & \frac{1}{z_0-L_7} \\
\frac{1}{z_1-L_0} & \frac{1}{z_1-L_1} & \frac{1}{z_1-L_2} & \frac{1}{z_1-L_3} & \frac{1}{z_1-L_4} & \frac{1}{z_1-L_5} & \frac{1}{z_1-L_6} & \frac{1}{z_1-L_7} \\
\hline
\frac{1}{z_2-L_0} & \frac{1}{z_2-L_1} & \frac{1}{z_2-L_2} & \frac{1}{z_2-L_3} & \frac{1}{z_2-L_4} & \frac{1}{z_2-L_5} & \frac{1}{z_2-L_6} & \frac{1}{z_2-L_7} \\
\frac{1}{z_3-L_0} & \frac{1}{z_3-L_1} & \frac{1}{z_3-L_2} & \frac{1}{z_3-L_3} & \frac{1}{z_3-L_4} & \frac{1}{z_3-L_5} & \frac{1}{z_3-L_6} & \frac{1}{z_3-L_7} \\
\hline
\frac{1}{z_4-L_0} & \frac{1}{z_4-L_1} & \frac{1}{z_4-L_2} & \frac{1}{z_4-L_3} & \frac{1}{z_4-L_4} & \frac{1}{z_4-L_5} & \frac{1}{z_4-L_6} & \frac{1}{z_4-L_7} \\
\frac{1}{z_5-L_0} & \frac{1}{z_5-L_1} & \frac{1}{z_5-L_2} & \frac{1}{z_5-L_3} & \frac{1}{z_5-L_4} & \frac{1}{z_5-L_5} & \frac{1}{z_5-L_6} & \frac{1}{z_5-L_7} \\
\hline
\frac{1}{z_6-L_0} & \frac{1}{z_6-L_1} & \frac{1}{z_6-L_2} & \frac{1}{z_6-L_3} & \frac{1}{z_6-L_4} & \frac{1}{z_6-L_5} & \frac{1}{z_6-L_6} & \frac{1}{z_6-L_7} \\
\frac{1}{z_7-L_0} & \frac{1}{z_7-L_1} & \frac{1}{z_7-L_2} & \frac{1}{z_7-L_3} & \frac{1}{z_7-L_4} & \frac{1}{z_7-L_5} & \frac{1}{z_7-L_6} & \frac{1}{z_7-L_7}
\end{array}\right]
\end{array}
$$

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[MB09] generates a large full Dyadic block then picks up

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- We propose to generate a small full Dyadic block and then generate other full Dyadic blocks by introducing $\oplus\Delta$ (we call it outer delta)
  - Whereas we call $\oplus\delta$ inner delta (since it defines the inner structure of a full Dyadic block

$$\begin{array}{cccc} & \oplus\Delta_1 & \oplus\Delta_2 & \oplus\Delta_3 \end{array}$$

$$\left[\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\right]$$

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Advantages

- Can generate the txn Dyadic matrix more flexibly,
  - $N=2^m$ can be closer to n+t
- Can remove the block-wise permutation and removal in key generation
  - Since they are already included by the parameter choice of $\oplus\Delta$

Remark

- Block-Wise Permutation is the same as changing Δ_j's appropriately.
- Removal of one block is the same as reducing the number of blocks by one and then changing Δ_j's appropriately

$$\begin{array}{cccc} & \oplus\Delta_1 & \oplus\Delta_2 & \oplus\Delta_3 \\ \left[\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\right. & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \left.\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\right] \end{array}$$

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By applying $\oplus\delta$ and $\oplus\Delta$ to the denominators

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- txn Dyadic matrix can be obtained with

$$z_1 = z_0 \oplus \delta_1 \quad z_2 = z_0 \oplus \delta_2 \quad z_3 = z_2 \oplus \delta_1 = z_0 \oplus \delta_1 \oplus \delta_2$$

$$L_1 = L_0 \oplus \delta_1 \quad L_2 = L_0 \oplus \delta_2 \quad L_3 = L_2 \oplus \delta_2 = L_0 \oplus \delta_1 \oplus \delta_2$$

$$L_t = L_0 \oplus \Delta_1 \quad L_{j \times t} = L_0 \oplus \Delta_j$$

- where

$$z_0, L_0, \delta_1, \delta_2 \cdots \delta_{\log_2 t}, \Delta_1, \Delta_2 \cdots \Delta_{(n/t)-1} \in GF(2^m)$$

are chosen at random while making all the z_i for $0 \le i < t$ and L_j for $0 \le j < n$ distinct.

Shuffle While Keeping Dyadic Structure

- With Column-Block-Wise Scalar Multiplication

$$\left[\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\cdots\begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix}\right]\begin{bmatrix}[1] & & & 0\\ & [p_1] & & \\ & & \ddots & \\ 0 & & & [p_{b-1}]\end{bmatrix}$$

where $p_i \in GF(q^m)\setminus\{0\}$ and $b = \dfrac{n}{t}$

GF(q^m) to GF(q)

$A, \gamma_i \in GF(q^m)$

$a_i \in GF(q)$

$\{\gamma_1, \gamma_2 \cdots \gamma_{m\text{-}1}\}$ is the dual basis of $GF(q^m)$

$$A = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{m-1} \end{bmatrix} = \begin{bmatrix} tr(\gamma_0 A) \\ tr(\gamma_1 A) \\ \vdots \\ tr(\gamma_{m-1} A) \end{bmatrix}$$

$$tr(A) = A + A^q + A^{q^2} + \cdots + A^{q^{m-1}}$$

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GF(q^m) to GF(q)

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$$\begin{bmatrix} A & B \\ B & A \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_{m-1} \end{bmatrix} & \begin{bmatrix} b_0 \\ \vdots \\ b_{m-1} \end{bmatrix} \\ \begin{bmatrix} b_0 \\ \vdots \\ b_{m-1} \end{bmatrix} & \begin{bmatrix} a_0 \\ \vdots \\ a_{m-1} \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix} a_0 & b_0 \\ b_0 & a_0 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} a_{m-1} & b_{m-1} \\ b_{m-1} & a_{m-1} \end{bmatrix} \end{bmatrix}$$

Shuffle While Keeping Dyadic Structure

- Multiplication of a Random Dyadic Matrix

$$\begin{bmatrix} \begin{bmatrix}\text{Random}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Random}\\ \text{Dyadic}\end{bmatrix} \\ \begin{bmatrix}\text{Random}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Random}\\ \text{Dyadic}\end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} \\ \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} & \begin{bmatrix}\text{Full}\\ \text{Dyadic}\end{bmatrix} \end{bmatrix}$$

$$= (HS)^T$$

Note: Dyadic X Dyadic = Dyadic

$$\begin{bmatrix} A & B \\ B & A \end{bmatrix}\begin{bmatrix} C & D \\ D & C \end{bmatrix} = \begin{bmatrix} AC+BD & AD+BC \\ BC+AD & BD+AC \end{bmatrix}$$

Attack on Quasi Dyadic [UL09]

- Exploits the fact that each column of each low of H'^T is $tr(f(L_i))$ where f() and L_i are unknown
- If one can make f(x)=const, $tr(f(L_i))$ becomes independent of L_i and all the coordinates in the same block of the low take the same value.
- On the contrary, by choosing r_i s.t.

$$r_i H'^T = ([h_{i,0}, \cdots, h_{i,0}], [h_{i,1}, \cdots, h_{i,1}], \cdots, [h_{i,b-1}, \cdots, h_{i,b-1}])$$

f(x) can be const with high probability

Attack on Quasi Dyadic [UL09]

$$r_i H'^T = (\boxed{[h_{i,0},\cdots,h_{i,0}]},[h_{i,1},\cdots,h_{i,1}],\cdots,[h_{i,b-1},\cdots,h_{i,b-1}])$$

These constants are linear combinations of tr(constant)

$$\begin{bmatrix} r_0 \\ r_1 \\ \vdots \\ r_{m-1} \end{bmatrix} H'^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} =$$

m(m-1) unknown in GF(q) so the uncertainty of (p_1 to p_{m-1}) is m(m-1)$\log_2$ q

$$\begin{bmatrix} h_{0,0} & \beta_{0,1} & \cdots & \beta_{0,m-1} \\ h_{1,0} & \beta_{1,1} & \cdots & \beta_{1,m-1} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m-1,0} & \beta_{m-1,1} & \cdots & \beta_{m-1,m-1} \end{bmatrix} \begin{bmatrix} [tr(\gamma_0 1),\cdots] & [tr(\gamma_0 p_1),\cdots] & \cdots \\ [tr(\gamma_1 1),\cdots] & [tr(\gamma_1 p_1),\cdots] & \cdots \\ \vdots & \vdots & \cdots \\ [tr(\gamma_{m-1} 1),\cdots] & [tr(\gamma_{m-1} p_1),\cdots] & \cdots \end{bmatrix}$$

Countermeasure Against The Attack on Quasi Dyadic

- m(m-1) unknown in GF(q) and hence the uncertainty of (p_1 to p_{m-1}) is m(m-1)$\log_2 q$ bits
- E.g.)

  For q=2, by making m>=10, the uncertainty becomes >=90 bits and this attack can be avoided

  (though this attack might be improved in future)

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Parameters for PKE

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m	t	Binary work factor	PK size (KB)	Plaintext / Ciphertext size	
				McEliece	Niederreiter
11	32	$2^{86.8}$	72.9KB	1696/2048 bits	233/352bits
12	41	$2^{128.5}$	216.5KB	3604/4096 bits	327/492bits

↓ QD (Parameters are not optimized)

m	t	n	p	l	Binary work factor	PK size (KB)	Plaintext/Ciphertext Size
							QD
16	64	2560	1	12	$2^{91.3}$	3.0KB	427/1024 bits
13	64	2048	2	16	$2^{90.2}$	1.9KB	406/832 bits
12	128	2400	1	8	$2^{90.7}$	1.3KB	716/1536 bits

Parameters for Digital Signature

m	t	BWF	PK size (KB)	Iteration *1	Signature Size *2,*3
14	13	$2^{80.7}$	360.0KB	$2^{32.5}$	182～214.5 bits
13	14	$2^{80.0}$	178.0KB	$2^{36.4}$	182～218.4 bits
12	15	$2^{88.2}$	86.0KB	$2^{40.3}$	180～220.3 bits

↓ QD (Parameters are not optimized)

m	t	n	Binary work factor	PK size (KB)	Iteration *1	Signature Size *2,*3
15	12	26528	$2^{84.0}$	48.2KB	$2^{32.5}$	180～212.5 bits
14	13	13316	$2^{83.4}$	22.4KB	$2^{36.4}$	182～218.4 bits
13	14	6736	$2^{82.9}$	10.4KB	$2^{40.3}$	182～222.3 bits

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Suitable Applications for Code-Based PKCs

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Code-Based PKCs Fit With Heterogeneous Network/Applications

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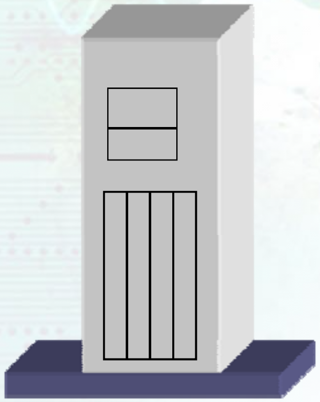
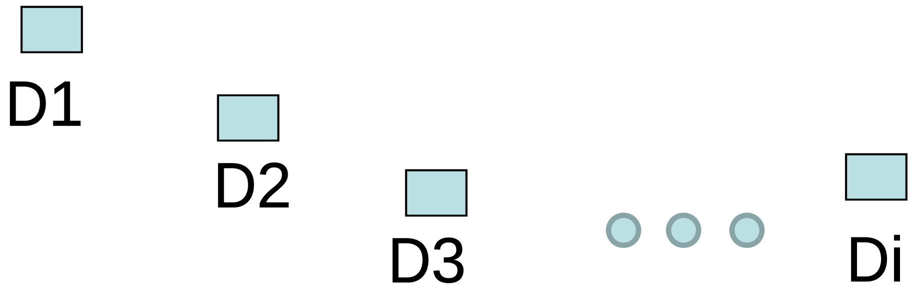


One side has high computational power



A lot of low cost and low computational power devices, such as RFIDs, sensors and SCADA devices

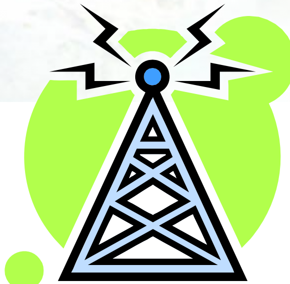
Since

•Encryption and signature verification consist mostly of xors and they are highly parallelizable.

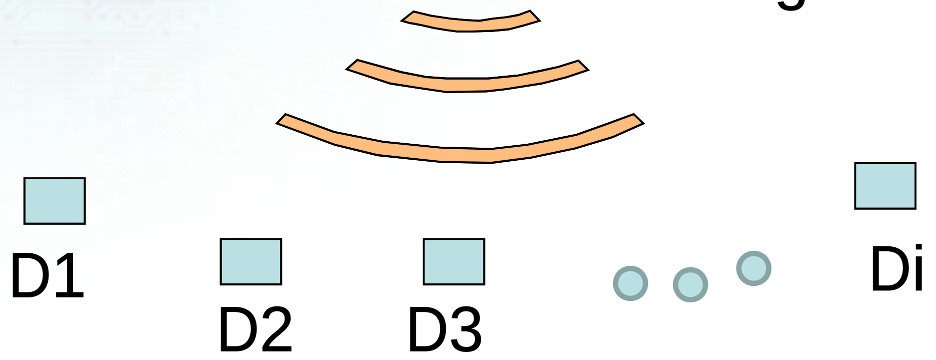
•They do not require heavy multi precision modular exponentiations

Broadcast Authentication

One (powerful) sender



broadcasts a message



a lot of (lightweight) receivers receive it and check its authenticity and data integrity.

Applications include:
- Transmission of mission critical commands and data to sensors and SCADA devices.
- Issue of disaster warning

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In emergency applications, low latency is crucial

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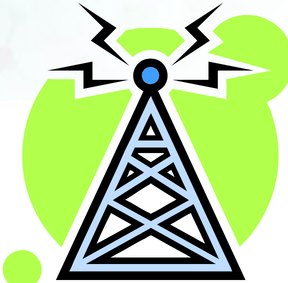
Especially → Against
- Earthquake
- Tsunami
- Flood
- Tornado
- Thunderbolt
- Fire

...

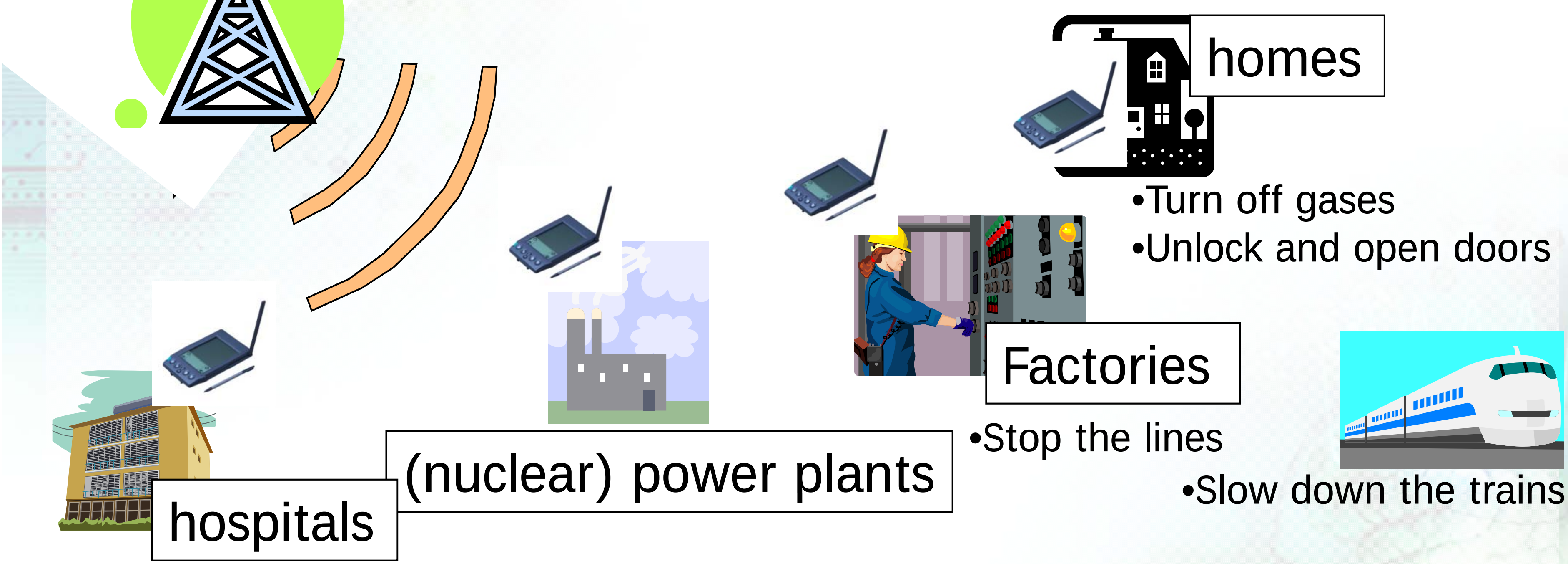
Disaster warning



D1 D2 D3 Di

Such warning receivers may be deployed

at home, critical infrastructures, and then take appropriate actions if a warning is verified.



In some cases, a few seconds are enough to mitigate serious damages, and hence delay is should be minimized while maintaining adequate security level.

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Comparison Among Solutions

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	MAC with one master key	MAC with pair-wise keys	TESLA (hash-chain and delayed auth)	Digital Signature
				Conventional (RSA, DSA, ECDSA)
Authenticity and Data Integrity	X*1	O	O	O
Computational Cost	O	O	O	X
Delay	O	X*2	X*3	X

*1: Crack of one device breaks it.
*2: Sender must broadcast a lot of MACs and each device must wait until his MAC is received.
*3: Verification key is released in the next time slot.

TESLA: Timed Efficient Stream Loss-tolerant Authentication

Comparison Among Solutions

	MAC with one master key	MAC with pair-wise keys	TESLA (hash-chain and delayed auth)	Digital Signature	
				Conventional (RSA, DSA, ECDSA)	Code-Based
Authenticity and Data Integrity	X*1	O	O	O	O
Computational Cost	O	O	O	X	O
Delay	O	X*2	X*3	X	O

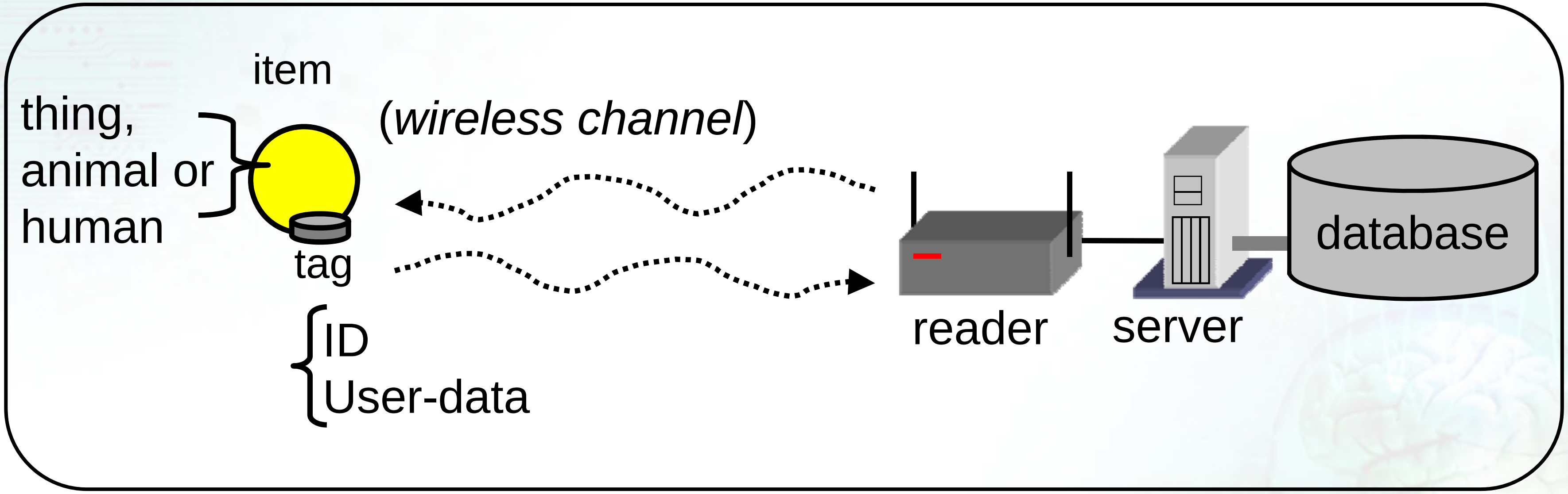
*1: Crack of one device breaks it.
*2: Sender must broadcast a lot of MACs and each device must wait until his MAC is received.
*3: Verification key is released in the next time slot.

TESLA: Timed Efficient Stream Loss-tolerant Authentication

Privacy Enhanced RFID

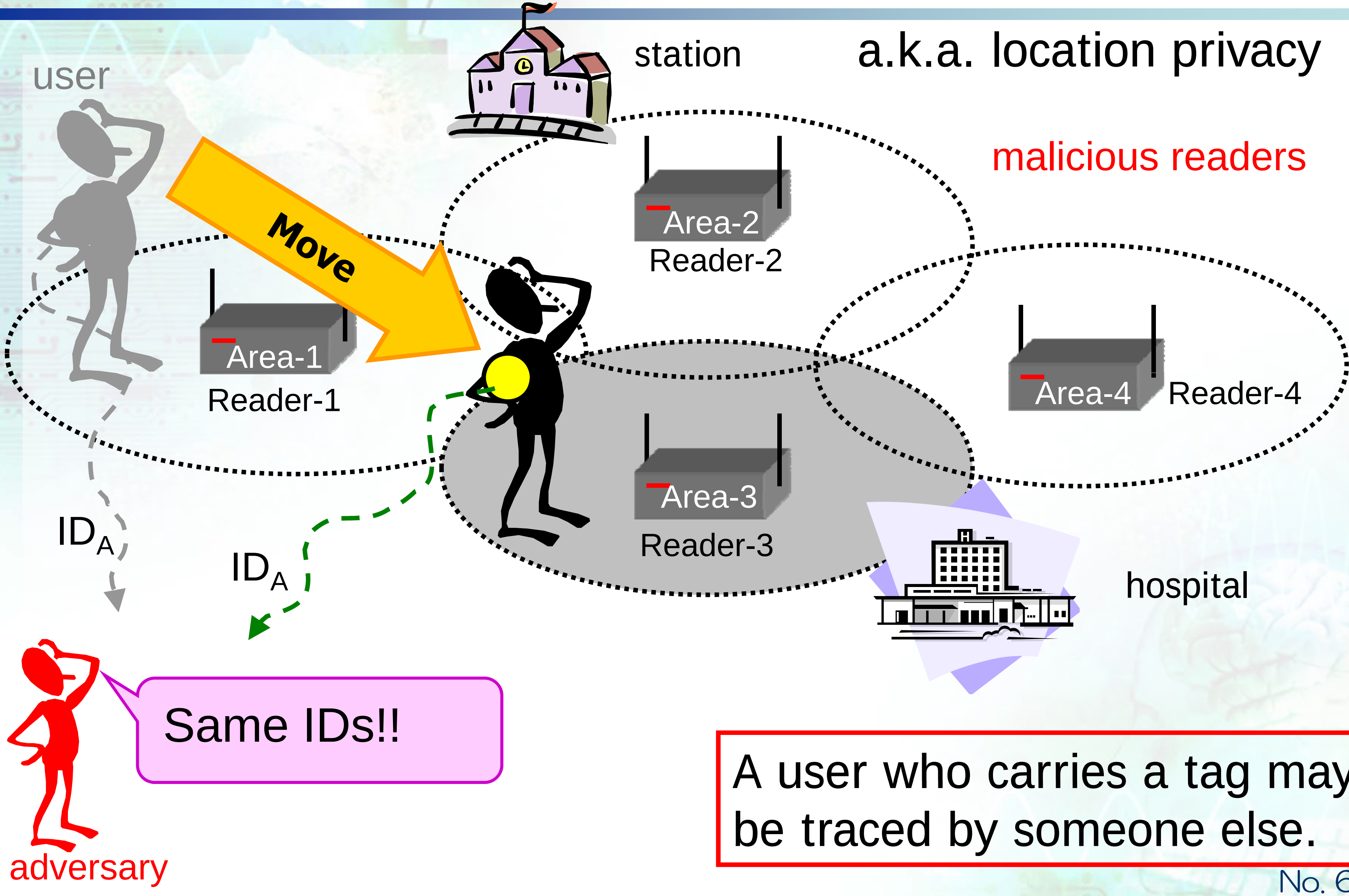
RFID: a tag used for reading/writing ID (and information) via wireless communication.

RFID system in general:



Applications: management of items, e.g. in supply chain

RFID (for long distance) requires privacy protection mechanism



A user who carries a tag may be traced by someone else.

Solutions Can Be Divided Into

- Tag disabling solutions
  - Manually removal or destruction
  - Kill command
- Temporally tag-disabling solutions
  - Faraday cage
  - Access password
  - Hash lock
  - Blocker tag
  - Mode switch
- Tag enabling solutions
  - Randomized hash lock [WSRE03]
  - HB+ [JW05] and its variants
  - Code-Based Unlinkable-ID [SKI06] [CKM+07]

Tag enabling solutions:
Enable RFID functionalities while providing unlinkability of IDs

Comparison

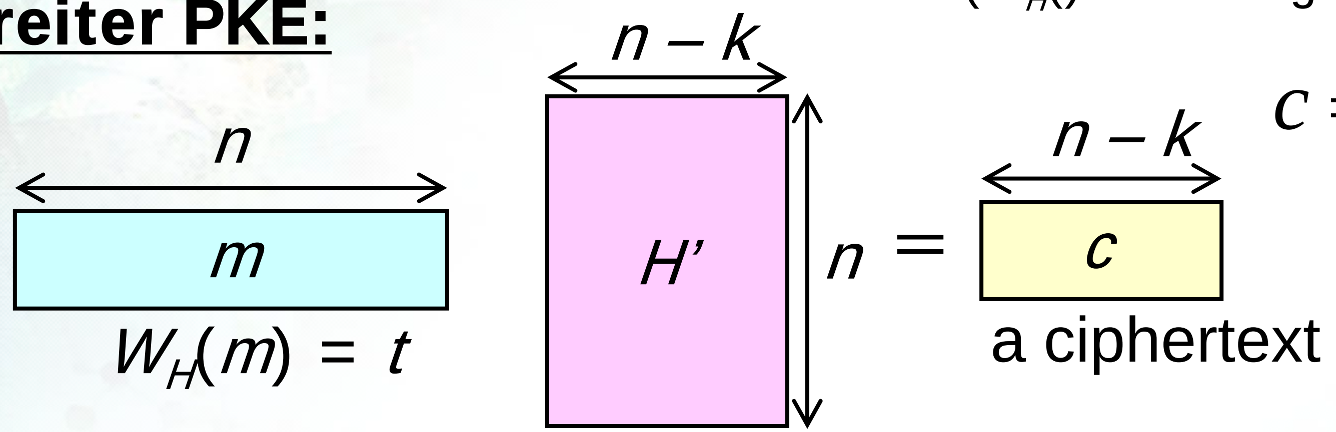
	Exhaustive search of IDs at the server	Unlinkability
Randomized hash lock, HB+ and its variants	Necessary (Tag identification cost depends on # of tags)	○
Code-Based Unlinkable-ID	Unnecessary (Tag identification cost is independent of # of tags)	○

Code-Based Unlikable-ID (1/3)

($W_H()$: Hamming weight)

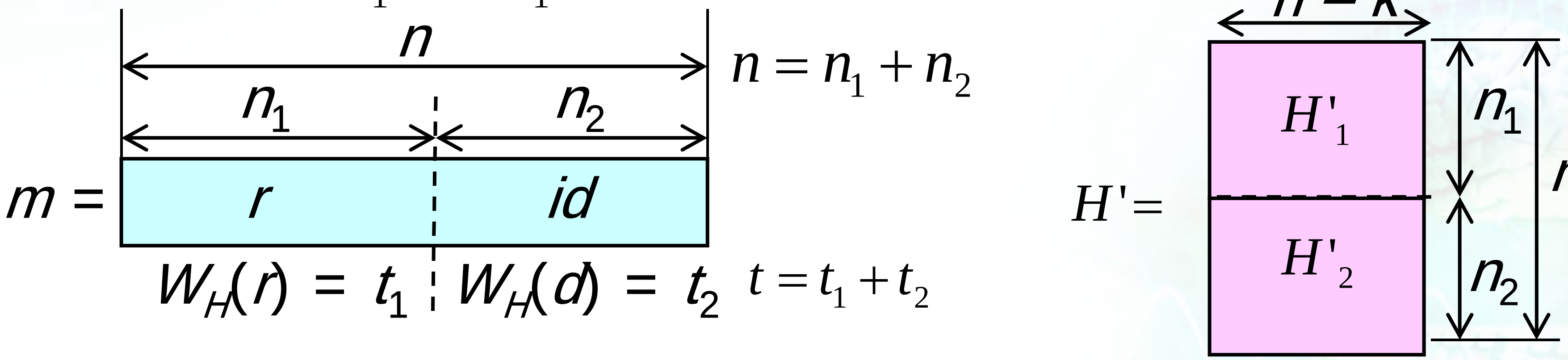
Niederreiter PKE:

$$c = mH'$$



Code-Based Unlinkable-ID:

$$a = n_1 / n = t_1 / t$$



(random num.) (ID)

Code-Based Unlinkable-ID (2/3)

$$\boxed{c} = \underbrace{\boxed{r} \times \boxed{H'_1}}_{c_1} \oplus \underbrace{\boxed{id} \times \boxed{H'_2}}_{c_2}$$

$$\boxed{c_1} \oplus \boxed{c_2}$$

Let each tag calculate this

Pre compute this at the server and assigns it to each tag

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Code-Based Unlinkable-ID (3/3)

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Query & Reply Phase

Tag (H'_1, c_2)

Reader (S, H, P)

Query

RNG( ) → r

$c_1 = r \times H'_1$

$c = c_1 \oplus c_2$

Unlinkable-ID $= C$

ID Resolve Phase

$r || id :=$ Decrypt(c)

This scheme provides unlinkability of IDs against passive attack. But by make this challenge-response type, this can also be secure against adaptive attack [CKM+07]

Conclusion (1/2)

- Code-based PKCs are suitable for
  - Heterogeneous Network/Applications, such as
    - Broadcast Authentication and Unlinkable-ID
  - for light weight devices such as
    - RFID, sensors, SCADA devices
- Since they
  - do not require heavy multi-precision modular exponentiation
  - can be executed mostly using only xors highly in parallel

Conclusion (2/2)

- Research themes left in this area include
  - Further reduction of PK sizes
  - New attacks (especially on QD)
  - New primitives/applications
  - Implementation and side-channel attacks
  - Provable security
  - etc.

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Thank you very much for your kind attention!!

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