

Observational and symbolic equivalence

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Joint work with Hubert Comon-Lundh and Stéphanie
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Goal: decision procedure for observational equivalence

Current results:

- ▶ Automatic testing equivalence verification from spi-calculus specifications.
[Durante, Sisto, Valenzano '03]
- ▶ Deciding framed bisimilarity.
[Huttel '02]
- ▶ Automated verification of selected equivalences for the applied pi-calculus.
[Blanchet, Abadi, Fournet '05]
- ▶ Symbolic bisimulation for the applied pi-calculus.
[Delaune, Kremer, Ryan '07]
- ▶ A method for proving observational equivalence.
[Cortier, Delaune '09]
- ▶ Deciding Security of Protocols against Off-line Guessing Attacks.
[Baudet '05]

The restrictions

A simple class of processes:

$$\begin{aligned} P_i ::= & \\ & 0 \\ & out(i, t).P_i \\ & in(i, x).if \Phi(\bar{x}) \text{ then } P_i \\ & (\nu\alpha)P_i \end{aligned}$$

A bounded number of sessions:

$$P_1 \mid \dots \mid P_n$$

Observations, bisimulations, traces

Observational equivalence $P \sim_o P'$:

$\forall C. C[P]$ behaves like $C[P']$

Static equivalence $T \sim T'$:

$\forall \pi_1, \pi_2. \pi_1[T] \downarrow = \pi_2[T] \downarrow \Leftrightarrow \pi_1[T'] \downarrow = \pi_2[T'] \downarrow$

Bisimulation $P \approx P'$:

$$\begin{cases} - \phi(P) \sim \phi(P') \\ - \forall a. (P \xrightarrow{a} Q \Leftrightarrow P' \xrightarrow{a} Q') \ \& \ Q \approx Q' \end{cases}$$

Trace equivalence $P \approx_t P'$:

$$\begin{cases} - \phi(P) \sim \phi(P') \\ - \forall w. (P \xrightarrow{w} Q \Leftrightarrow P' \xrightarrow{w} Q') \ \& \ \phi(P') \sim \phi(Q') \end{cases}$$

From concrete to symbolic traces

Concrete transitions: $E = (P_1 | \dots | P_n, S, \sigma)$

▶ $P_i \xrightarrow{in(i,x)} P'_i$ if $\Phi(\bar{x})$ then P'_i

▶ $P_i \xrightarrow{out(i,t)} P'_i$

From concrete to symbolic traces

Concrete transitions: $E = (P_1 | \dots | P_n, S, \sigma)$

- ▶ $P_i \xrightarrow{in(i,x)} \text{if } \Phi(\bar{x}) \text{ then } P'_i$
 $\forall t \text{ s.t. } S \vdash t :$
 - ▶ $E \xrightarrow{in(i,x)} (P_1 | \dots | P'_i | \dots | P_n, S \cup t, \sigma \uplus \{x \mapsto t\}),$
if $\sigma \uplus \{x \mapsto t\} \models \Phi$
 - ▶ $E \xrightarrow{fail} (\perp, \perp, \perp),$ otherwise
- ▶ $P_i \xrightarrow{out(i,t)} P'_i$

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 - ▶ $E \xrightarrow{out(i,t)} (P_1 | \dots P'_i \dots | P_n, S \cup t\sigma, \sigma)$

From concrete to symbolic traces

$R_A(a, b) = \nu n. out(enc(\langle a, n \rangle, pub(b))). in(enc(\langle a, n \rangle, pub(a))). ok$

$R_B(a, b) = in(enc(\langle a, x \rangle, pub(b))). out(enc(\langle a, x \rangle, pub(a))). 0$

From concrete to symbolic traces

$R_A(a, b) = \nu n. \text{out}(\text{enc}(\langle a, n \rangle, \text{pub}(b))). \text{in}(\text{enc}(\langle a, n \rangle, \text{pub}(a))). \text{ok}$

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Sessions : $R_A(a, b) | R_B(a, b) | R_A(a, c) | R_B(a, c)$

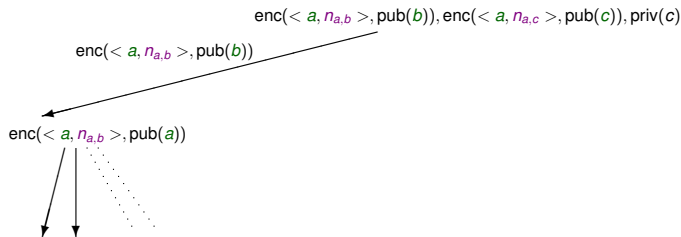
$\text{enc}(\langle a, n_{a,b} \rangle, \text{pub}(b)), \text{enc}(\langle a, n_{a,c} \rangle, \text{pub}(c)), \text{priv}(c)$

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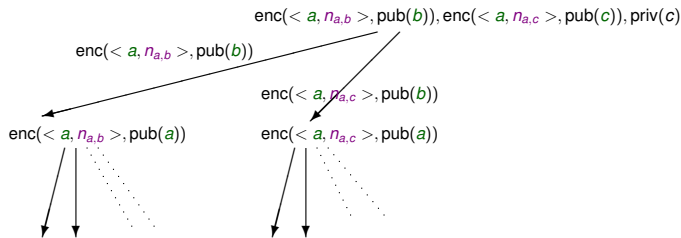


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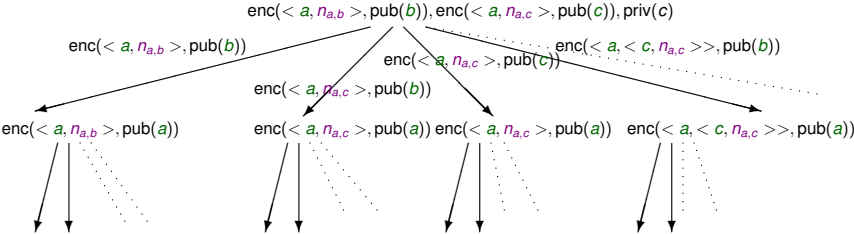


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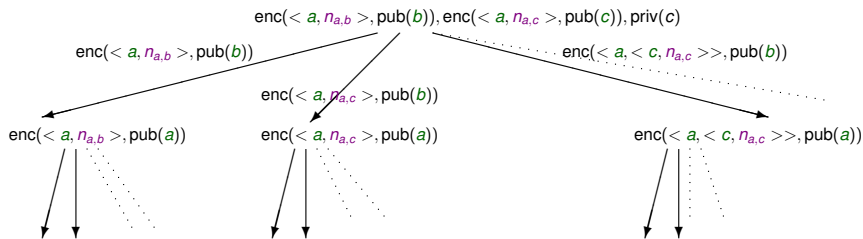


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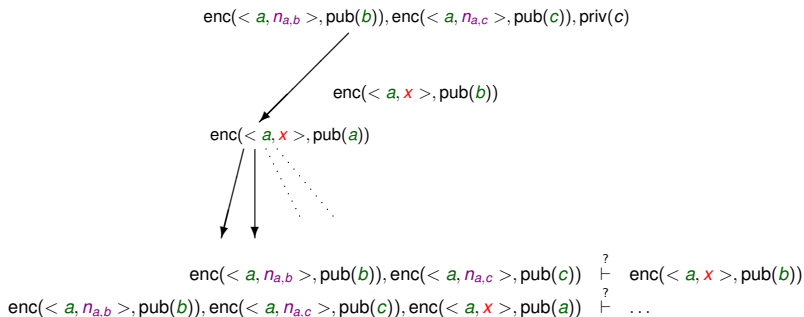


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From concrete to symbolic traces

Symbolic transitions: $E = (P_1 | \dots | P_n, S, \mathcal{C})$

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 - ▶ $E \xrightarrow{in(i,x)} (P_1 | \dots | P'_i | \dots | P_n, S \cup x, \mathcal{C} \wedge S \stackrel{?}{\vdash} x \wedge \Phi(\bar{x}))$,
if $\mathcal{C} \wedge S \stackrel{?}{\vdash} x \wedge \Phi(\bar{x})$ is satisfiable
 - ▶ $E \xrightarrow{fail} (\perp, \perp, \perp)$, otherwise
- ▶ $P_i \xrightarrow{out(i,t)} P'_i$
 - ▶ $E \xrightarrow{out(i,t)} (P_1 | \dots | P'_i | \dots | P_n, S \cup t, \mathcal{C})$

Trace equivalence

Concrete $P \approx_c P$:

$\forall w.$

$$P \xrightarrow{w} (P', S, \sigma) \Leftrightarrow Q \xrightarrow{w} (Q', S', \sigma') \ \& \\ S \sim S'$$

Symbolic $P \approx_s P$:

$\forall w.$

$$P \xrightarrow{w} (P', S, C) \Leftrightarrow Q \xrightarrow{w} (Q', S', C') \ \& \\ \forall \sigma \in \text{Sol}(C) \exists \sigma' \in \text{Sol}(C'). S\sigma \sim S'\sigma' \ \& \\ \forall \sigma' \in \text{Sol}(C') \exists \sigma \in \text{Sol}(C). S\sigma \sim S'\sigma'$$

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Equivalence of deducibility constraint systems

$$\mathcal{C} = \left\{ \begin{array}{l} S_1 \quad ? \quad x_1 \\ \vdots \\ S_n \quad ? \quad x_n \\ S \quad \Phi(\bar{x}) \end{array} \right. \quad \mathcal{C}' = \left\{ \begin{array}{l} S'_1 \quad ? \quad x_1 \\ \vdots \\ S'_n \quad ? \quad x_n \\ S' \quad \Phi'(\bar{x}) \end{array} \right.$$

$\mathcal{C} \approx \mathcal{C}'$:

- ▶ $\forall \sigma \in \mathbf{Sol}(\mathcal{C}) \exists \sigma' \in \mathbf{Sol}(\mathcal{C}'). S\sigma \sim S'\sigma'$
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Equivalence of deducibility constraint systems

$$C = \left\{ \begin{array}{l} S_1 \quad ? \quad u_1 \\ \vdots \\ S_n \quad ? \quad u_n \\ S \end{array} \right. \quad C' = \left\{ \begin{array}{l} S'_1 \quad ? \quad u'_1 \\ \vdots \\ S'_n \quad ? \quad u'_n \\ S' \end{array} \right.$$

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$$C = \left\{ \begin{array}{l} a \quad ? \quad x \\ a, x, \{a\}_{\{a\}_k}, \{x\}_k \end{array} \right. \quad C' = \left\{ \begin{array}{l} a \quad ? \quad x \\ a, x, \{a\}_{\{\langle a, a \rangle\}_k}, \{x\}_k \end{array} \right.$$

Equivalence of deducibility constraint systems

$$\mathcal{C} = \begin{cases} S_1 & \vdash & u_1 \\ \dots & & \\ S_n & \vdash & u_n \\ S & & \end{cases} \quad \mathcal{C}' = \begin{cases} S'_1 & \vdash & u'_1 \\ \dots & & \\ S'_n & \vdash & u'_n \\ S' & & \end{cases}$$

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Procedure:

- ▶ $\mathcal{C} \rightsquigarrow (\mathcal{C}_1, S_1), \dots, (\mathcal{C}_n, S_n)$ and $\mathcal{C}' \rightsquigarrow (\mathcal{C}'_1, S'_1), \dots, (\mathcal{C}'_m, S'_m)$
- ▶

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- ▶ $\mathcal{C} \approx \mathcal{C}' \Leftrightarrow$
 - ▶ $\forall i \exists j. S_i \sim S'_j$
 - ▶ $\forall j \exists i. S_i \sim S'_j$

Elements of a solution

$$c = \left\{ a, x, \{a\}_{\{a\}_k}, \{x\}_k \right. \quad a \stackrel{?}{\vdash} x$$
$$c' = \left\{ a, x, \{a\}_{\{<a,a>\}_k}, \{x\}_k \right. \quad a \stackrel{?}{\vdash} x$$

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- ▶ Guess equalities among subterms: $\theta_1, \dots, \theta_n$ and $\theta'_1, \dots, \theta'_m$
- ▶ $C \rightsquigarrow C\theta_1, \dots, C\theta_n$ and $C' \rightsquigarrow C'\theta'_1, \dots, C'\theta'_m$

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- ▶ Guess equalities among subterms: $\theta_1, \dots, \theta_n$ and $\theta'_1, \dots, \theta'_m$
- ▶ $C \rightsquigarrow C\theta_1, \dots, C\theta_n$ and $C' \rightsquigarrow C'\theta'_1, \dots, C'\theta'_m$
- ▶ Does not work (see example 2 next)

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Equalities must not be guessed in an arbitrary way, but linked to a proof

Constraint solving rules that...

- ▶ $\mathcal{C} \rightsquigarrow^* \mathcal{C}_1, \dots, \mathcal{C}_n$
- ▶ Do not miss any solution σ of \mathcal{C}
- ▶ Do not miss any proof π in \mathcal{C}_σ
- ▶ Do compute all and only equalities that are needed for π

Local theories

A deduction system is *local* if, whenever v is deducible from H , then there is a proof of $H \vdash v$ whose all intermediate steps are either subterms of H or subterms of v

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Example:

$$\frac{\text{enc}(a, \langle b, c \rangle) \quad \frac{b \quad c}{\langle b, c \rangle}}{a}$$
$$\frac{\text{enc}(a, b) \quad \frac{\frac{b \quad c}{\langle b, c \rangle}}{b}}{a}$$

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$$\frac{\begin{array}{ccc} & H & \\ \vdots & & \vdots \\ \hline v_1 & \dots & v_n \\ \hline & v & \end{array}}{v}$$

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$$v_1, \dots, v_n \in \text{St}(H, v)$$

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$v_1, \dots, v_n \in \text{St}(H)$ (for decompositions)

Locality, decompositions and constraint solving rules

Locality:

equalities are triggered by decompositions \implies

solved forms are systems with only compositional proofs

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Minimal unsolved constraint: $S_1 \stackrel{?}{\vdash}_c w_1, \dots, S_i \stackrel{?}{\vdash}_c w_i, S \stackrel{?}{\vdash} v, \dots$

► Last inference rule
$$\frac{u_1 \dots u_n}{u}$$

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Minimal unsolved constraint: $S_1 \vdash_c^? w_1, \dots, S_i \vdash_c^? w_i, S \vdash^? v, \dots$

- ▶ Last inference rule
$$\frac{u_1 \dots u_n}{u}$$
- ▶ Constraint solving rule (decomposition)

$$S \vdash^? v \rightsquigarrow \begin{cases} S \vdash^? v_1, \dots, S \vdash^? v_n \\ v = u, v_1 = u_1, \dots, v_n = u_n, \\ \text{where } v_1, \dots, v_n \in \text{St}(H) \end{cases}$$

A way to represent all intruder actions

- ▶ $\mathcal{C} \rightsquigarrow \mathcal{C}_1, \dots, \mathcal{C}_n$ and $\mathcal{C}' \rightsquigarrow \mathcal{C}'_1, \dots, \mathcal{C}'_n$

A way to represent all intruder actions

- ▶ $\mathcal{C} \rightsquigarrow \mathcal{C}_1, \dots, \mathcal{C}_n$ and $\mathcal{C}' \rightsquigarrow \mathcal{C}'_1, \dots, \mathcal{C}'_n$
- ▶ $\mathcal{C} \approx \mathcal{C}'$ iff
 - ▶ $\forall i \exists j. \mathcal{S}(\mathcal{C}_i) \sim \mathcal{S}(\mathcal{C}'_j)$
 - ▶ $\forall j \exists i. \mathcal{S}(\mathcal{C}_i) \sim \mathcal{S}(\mathcal{C}'_j)$

Questions

Design: Are simple processes enough for what we need?
If not, can we extend symbolic methods to a larger class?

- Analysis:**
- ▶ Static equivalence: more theories? finer relations?
 - ▶ Locality: what theories are local? how to handle non-local theories?

Proof theory: are the restrictions that come from the application necessary for decidability?