Protocol composition logic
symbolic model, computational model,
and applications

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Second lecture on PCL will be given by Anupam Datta
Protocol Composition Logic (PCL)

Summary

PCL is a logic for proving security properties of network protocols. Two central results for PCL are a set of composition theorems and a computational soundness theorem. In contrast to traditional folk wisdom in computer security, the composition theorems allow proofs of complex protocols to be built up from proofs of their constituent sub-protocols. The computational soundness theorem guarantees that for a class of security properties and protocols, axiomatic proofs in PCL carry the same meaning as reduction-style cryptographic proofs. Tool implementation efforts are also underway. PCL and a complementary model-checking method have been successfully applied to a number of Internet, wireless and mobile network security protocols developed by the IEEE and IETF Working Groups. This work identified serious security vulnerabilities in the IEEE 802.11i wireless security standard and the IETF GDOI standard. The suggested fixes have been adopted by the respective standards bodies.

PCL has been the topic of invited talks at premier venues including AS.IA’01, MFPS’03, ICALP’05, LCC’06, and ASIAN’06. It has been taught in security courses at a number of universities including Aachen, CMU, Penn, Stanford, and Texas. Three papers on this work have been invited to special issues of journals, which are compilations of the best papers presented at the respective venues.

The following paper and set of slides provides an overview of this project. For further details, please read the other papers included below.


Also see the model-checking page for related projects.

http://crypto.stanford.edu/protocols/
Goals

• PCL is an evolving research framework for investigating this basic question:
  
  Is it possible to prove security properties of current practical protocols using compositional, direct reasoning that does not mention the actions of the attacker?

• Direct reasoning
  
  If Alice creates and sends a nonce \( n \) and later receives Bob’s signature on \( \langle n,m \rangle \), then Bob signed \( \langle n,m \rangle \) after Alice created and sent the nonce.
Goals

• Combine the advantages of BAN
  – Annotate programs with assertions
  – High-level direct reasoning
  – No explicit reasoning about attacker

• With accepted protocol semantics
  – Set of roles executed concurrently by principals
  – Attacker controls the network
  – *Eventually*: symbolic and computational semantics
Goals

• Case studies of IETF, IEEE standard protocols
  – Eventually: proofs of some kind for all major widely used network security protocols
  – SSL/TLS, WPA2, IKEv2, Kerberos (PK-init, ...), ...

• Even if
  – Some of these protocols only have “weak” security guarantees under “reasonable” assumptions about the crypto primitives they use
Non-Goal

- Full formal proofs in this decade
  - PCL has axioms, proof rules about protocol steps
    \[ \text{[ new } n\text{]}_P \text{ “Knows}(P,n)\text{”} \]
  - Includes a Rule of Consequence [Hoare...]
    \[ \varphi \text{ [ actions ]}_P \psi \quad \psi \supset \theta \]
    \[ \varphi \text{ [ actions ]}_P \theta \]
    but does not contain specific rules for \( \psi \supset \theta \)

Someone can do this later if everything else works out
PCL has been a team effort

- Collaborators
  M. Backes, A. Datta, A. Derek, N. Durgin, C. He,
  R. Kuesters, D. Pavlovic, A. Ramanathan, A. Roy,
  A. Scedrov, V. Shmatikov, M. Sundararajan, V. Teague,
  M. Turuani, B. Warinschi, ...

Science is a social process
Protocol analysis spectrum

- Hand proofs
- Poly-time calculus
- Multiset rewriting with $\exists$
- Spi-calculus
- Athena
- Paulson
- NRL
- Strand spaces
- BAN logic
- Protocol logic
- Model checking
- FDR
- Mur$\varphi$
Protocol composition logic

- Alice’s information
  - Private data
  - Protocol
  - Sends and receives

Honest Principals, Attacker

Reason from local information

Private Data

Protocol
Challenge-Response Protocol

Running example for a number of slides.
Protocol logic: Actions

send m; send message m
receive x; receive a message into variable x
new n; generate new nonce n

• A program is a sequence of actions

\[
\text{InitCR}(A, B) = \left[ \begin{array}{l}
\text{new } m; \\
\text{send } A, B, \langle m, A \rangle; \\
\text{receive } B, A, n, \text{sig}_B \{ \text{r}, m, n, A \}; \\
\text{send } A, B, \text{sig}_A \{ \text{i}, m, n, B \}; \\
\end{array} \right]_A
\]

\[
\text{RespCR}(B) = \left[ \begin{array}{l}
\text{receive } A, B, \langle m, A \rangle; \\
\text{new } n; \\
\text{send } B, A, \langle n, \text{sig}_B \{ \text{r}, m, n, A \} \rangle; \\
\text{receive } A, B, \text{sig}_A \{ \text{i}, m, n, B \}; \\
\end{array} \right]_B
\]

Example send action is send m’ where m’ = ⟨A, B, ⟨m, A⟩⟩ includes source and destination
Symbolic Attacker

• Controls complete network
  – Can read, remove, inject messages

• Fixed set of operations on terms
  – Pairing, Projection
  – Encryption with known key
  – Decryption with known key
  – ...

• Represent attacker
  – by large set of attacker programs that can do these actions (c.f. “penetrator strands”)
Execution Model

• Initial Configuration
  – Set of principals and keys assigned to them
  – Assignment of \( \geq 1 \) role to each principal

• Run
  – Interleaving of actions of honest principals and attacker, starting from initial configuration

\[
\begin{array}{cccccc}
\text{A} & \text{send \{x\}_B} & \downarrow & \text{receive \{x\}_B} & \text{receive \{z\}_B} & \text{Position in run} \\
\downarrow & \text{new x} & \downarrow & \uparrow & \uparrow & \\
\text{B} & \text{send \{z\}_B} & \downarrow & \text{new z} & \text{send \{z\}_B} & \\
\text{C} & & & & & \\
\end{array}
\]
Formulas true at a position in run

- **Action formulas**
  \[ a ::= \text{Send}(P,t) \mid \text{Receive}(P,t) \mid \text{New}(P,t) \]
  \[ \mid \text{Decrypt}(P,t) \mid \text{Verify}(P,t) \]

- **Formulas**
  \[ \varphi ::= a \mid \text{Has}(P,t) \mid \text{Fresh}(P,t) \mid \text{Honest}(N) \]
  \[ \mid \text{Contains}(t_1, t_2) \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \exists x \varphi \]
  \[ \mid a < a' \]

- **Modal formula**
  \[ \varphi [ \text{actions} ]_P \psi \]

- **Example**
  \[ \text{Has}(X, \text{secret}) \supset (X = A \lor X = B) \]

Speccifies secrecy
Challenge-Response Property

Specifying authentication for Responder

\[
\text{CR} \models \text{true} \ [\text{RespCR}(B) \land \text{Honest}(A)] \supset \big( \\
\quad \text{Send}(A, \langle A, B, \langle m, A \rangle \rangle) < \text{Receive}(B, \langle A, B, \langle m, A \rangle \rangle) \land \\
\quad \text{Receive}(B, \langle A, B, \langle m, A \rangle \rangle) < \text{Send}(B, \langle B, A, \langle n, \text{sig}_B \{\text{“r”}, m, n, A \} \rangle \rangle) \land \\
\quad \text{Send}(B, \langle B, A, \langle n, \text{sig}_B \{\text{“r”}, m, n, A \} \rangle \rangle) < \text{Receive}(A, \langle B, A, \langle n, \text{sig}_B \{\text{“r”}, m, n, A \} \rangle \rangle) \land \\
\quad \text{Receive}(A, \langle B, A, \langle n, \text{sig}_B \{\text{“r”}, m, n, A \} \rangle \rangle) < \text{Send}(A, \langle A, B, \langle \text{sig}_A \{\text{“i”}, m, n, B \} \rangle \rangle) \land \\
\quad \text{Send}(A, \langle A, B, \langle \text{sig}_A \{\text{“i”}, m, n, B \} \rangle \rangle) < \text{Receive}(B, \langle A, B, \langle \text{sig}_A \{\text{“i”}, m, n, B \} \rangle \rangle)
\big)
\]

“Actions in Order”

Authentication as “matching conversations” [Bellare-Rogaway93]
Proof System

• Prove security properties of useful protocols
• Axioms
  – Simple formulas about actions, etc.
• Inference rules
  – Proof steps
• Theorem
  – Formula obtained from axioms by application of inference rules
Core concept: Honesty

- A principal $X$ is honest in run $R$ if
  - Intuitively,
    “$X$ only does what $X$ is supposed to do”
  - More precisely
    The actions of $X$ in $R$ are precisely an interleaving of initial segments of traces of a set of roles of the protocol.

We assume that protocols do not reveal pre-assigned keys of any principal. Certain axioms and rules are sound only under this assumption. These axioms and rules can be dropped and replaced if the assumption is dropped.
Sample axioms

- **Actions**
  \[ \text{true } [\text{send } m ]_p \text{ Send}(P,m) \]

- **Public key encryption**
  \[ \text{Honest}(X) \land \text{Decrypt}(Y, \text{enc}_X\{m\}) \supset X=Y \]

- **Signature**
  \[ \text{Honest}(X) \land \text{Verify}(Y, \text{sig}_X\{m\}) \supset \text{Sign}(X, \text{sig}_X\{m\}) \]
Authentication for CR Responder – part 1

\[
\text{InitCR}(A, B) = [ \\
\text{new } m; \\
\text{send } A, B, \langle m, A \rangle; \\
\text{receive } B, A, \langle n, \text{sig}_B\{"r", m, n, A\} \rangle; \\
\text{send } A, B, \text{sig}_A\{"i", m, n, B\}; \\
]_A
\]

\[
\text{RespCR}(B) = [ \\
\text{receive } A, B, \langle m, A \rangle; \\
\text{new } n; \\
\text{send } B, A, \langle n, \text{sig}_B\{"r", m, n, A\} \rangle; \\
\text{receive } A, B, \text{sig}_A\{"i", m, n, B\}; \\
]_B
\]

1. B reasons about his own action
\[
\text{CR } \vdash \text{ true } [ \text{RespCR}(B) ]_B \text{Verify}(B, \text{sig}_A\{"i", m, n, A\})
\]

2. Use signature axiom
\[
\text{CR } \vdash \text{ true } [ \text{RespCR}(B) ]_B \text{Honest}(A) \Rightarrow \text{Sign}(A, \text{sig}_A\{"i", m, n, A\})
\]
Proving Invariants

• We want to prove
  \[ \Gamma \equiv \text{Honest}(A) \supset \varphi, \]
  where \( \varphi \equiv \)
  \[ (\text{Sign}(A, \text{sig}_A(“i”, m, n, B)) \rightarrow \text{Receive}(A, \langle n, \text{sig}_B(“r”, m, n, A) \rangle)) \]

• “\( \varphi \) holds at all pausing states of all honest roles”
  – protocol segment: subsequence of honest party actions between pausing states
  – Picture of when invariant \( \varphi \) holds:

  \( \varphi \rightarrow \text{actions of A} \rightarrow \varphi \rightarrow \text{actions of B} \rightarrow \varphi \rightarrow \text{attacker actions} \rightarrow \varphi \rightarrow \text{actions of B} \rightarrow \varphi \rightarrow \ldots \)
Why is this an invariant of CR?

\[
\text{InitCR}(A, B) = [
\begin{align*}
&\text{new } m; \\
&\text{send } A, B, \langle m, A \rangle; \\
&\text{receive } B, A, \langle n, \text{sig}_B\{r\}, m, n, A \rangle; \\
&\text{send } A, B, \text{sig}_A\{i\}, m, n, B; \\
\end{align*}
\]
\[]_A

\[
\text{RespCR}(B) = [
\begin{align*}
&\text{receive } A, B, \langle m, A \rangle; \\
&\text{new } n; \\
&\text{send } B, A, \langle n, \text{sig}_B\{r\}, m, n, A \rangle; \\
&\text{receive } A, B, \text{sig}_A\{i\}, m, n, B; \\
\end{align*}
\]
\[]_B

• Honest behavior
  – One or more instances of these two roles

• Property of each role
  – If A signs \(\text{sig}_A\{i\}, m, n, B\)
    A must be executing InitCR role
    A previously received \(\langle B, A \langle n, \text{sig}_X\{r\}, m, n, A \rangle \rangle\);
Honesty Rule

• Rule for establishing invariants:
  – Prove $\varphi$ holds when threads are started
  – Prove, for all protocol segments, if $\varphi$ held at the beginning, it holds at the end

We have formulated more than one honesty rule, plus secrecy induction. Eventually: we would like to unify these rules.
Authentication for CR Responder – part 2

• So far
  – CR |- true [ RespCR(B) ]_B \text{Sign}(A, \text{sig}_A\{“i”, m, n, A\})

• Use invariant $\Gamma$ to prove:
  – CR |- true [ RespCR(B) ]_B \text{Receive}(A, n, \text{sig}_B\{“r”, m, n, A\})

• Reason from B’s point of view to prove:
  – CR |- true [ RespCR(B) ]_B \text{FirstSend}(B, n, \langle n, \text{sig}_B\{“r”, m, n, A\rangle\rangle)

• Apply Nonce freshness axiom to prove:
  – CR |- true [ RespCR(B) ]_B
    \text{Receive}(A, \langle n, \text{sig}_B\{“r”, m, n, A\} \rangle) < \text{Send}(B, \text{sig}_B\{“r”, m, n, A\})

• Additional similar steps complete the proof
Sample PCL studies

• Wireless 802.11i
  – Model checking to find errors, improve
  – PCL proof of correctness, including TLS

• Kerberos
  – Including variants “PK-Init” and “DH-init”

• Extensible Authentication Protocol (EAP)
  – Model check to find errors, improve
  – PCL proof of correctness, identify subtleties

• Mesh Security Architecture (IEEE 802.11s)
  – *Motorola group* added some axioms, found problems, identified invariants, proved correctness
Protocol composition logic

- Protocol
- Honest Principals, Attacker

- Alice’s information
  - Protocol
  - Private data
  - Sends and receives

Reason from local information
Principal may execute many roles
Some Details

• Protocol
  – Given by a set of roles

• Role
  – Program for one participant in protocol
  – Example:
    • InitCR(A,B) : A initiates Challenge-Response with B

• Principal
  – Agent, associated with a key pair, signing key, and/or symmetric key

• Thread
  – A role, instantiated and executed by a principal
  – Semantically: Principal, role instance, unique thread ID
Some Details

- Notation in PCL papers
  - Threads X, Y, Z, ... Executed by principals \( \hat{X}, \hat{Y}, \hat{Z}, \ldots \)
  - Some abuse of notation for readability (order-sorted algebra) ...

\[
\text{InitCR}(A, X) = [ \ldots \text{sig}_A\{"i", m, n, B\} \ldots ]_A
\]

Principal, key associated with principal, thread
Formulas

- **Action Formulas**
  \[ a ::= \text{Send}(X,m) \mid \text{Receive}(X,m) \mid \ldots \]
  principal X sends message m in thread X

- **Formulas**
  \[ \varphi ::= a \mid a < a' \mid \text{Has}(X,m) \mid \text{Fresh}(X,m) \mid \text{Honest}(N) \mid \ldots \]
  an action a happens before an action a’
  principal N is honest

- **Modal formulas**
  \[ \Psi ::= \varphi [ \text{actions} ]_X \psi \]
  if \( \varphi \) before, then after thread X completes actions, \( \psi \)
Semantics

• **Protocol Q**
  – Provides set of roles (e.g., initiator, responder)

• **Run R of Q**
  – Sequence of actions by principals following roles, plus attacker

• **Satisfaction**
  – \( Q, R \models \theta \lbrack \text{actions} \rbrack_p \varphi \)
    If some role of P in R does exactly actions, starting from state where \( \theta \) is true, then \( \varphi \) is true in state after actions completed

  – \( Q \models \theta \lbrack \text{actions} \rbrack_p \varphi \)
    For all runs R of Q
Formula $\varphi$ satisfied by protocol $Q$ at run $R$

- Defined by induction on formula $\varphi$
  
  $Q,R \models \text{Send}(X,m)$ if thread $X$ sent $m$ in $R$ ...
  
  $Q,R \models \text{Honest}(\hat{X})$ if $\hat{X}$ is an honest principal in the initial configuration of $R$ and $R|_{\hat{X}}$ is an interleaving of basic sequences of instances of roles of $Q$

  ...

  $Q,R \models \varphi [P]_X \psi$ if for all partitions $R = R_0R_1R_2$ and all substitutions $\sigma$, if $Q,R_0 \models \sigma\varphi$ and $\sigma'$ matches $P$ to $R_1|_X$ then $Q,R_2 \models (\sigma\bullet\sigma')\psi$

The first substitution is a symbolic environment giving values to variables.
The second accounts for how $P$ uses variables and the way operations in $P$ bind variables in $\psi$. 
Core concept: $[ \ldots ]_X$

$[a_1 \ a_2 \ a_3 \ \ldots \ ]_X \psi \ \text{vs} \ \ a_1^X < a_2^X < a_3^X < \ldots \ \supset \ \psi$

where if $a_i = \text{send} \ m$ then $a_i^X = \text{Send}(X,m)$

• Modal form
  – Thread X did $a_1 \ a_2 \ a_3 \ \ldots \ \text{in this order, with no other actions interleaved}$

• Non-modal form
  – Thread X did $a_1 \ a_2 \ a_3 \ \ldots \ \text{in order, but might have done some other things too in between or after}$
Proving absence of actions

• Some axioms
  \[
  \text{Start } [\ ]_X \rightarrow a^X \\
  \rightarrow a^X [b]_X \rightarrow a^X
  \]
  provided a, b do not unify

• Relevant proof rule
  \[
  \frac{\varphi [ S ]_X \psi \quad \psi [ T ]_X \theta}{\varphi [ ST ]_X \theta}
  \]

• In contrast,
  \[
  a_1^X < a_2^X < a_3^X \supset \rightarrow b^X
  \]
  is invalid
Honesty rule

(\forall \text{roles } R \text{ of } Q.
\forall \text{ protocol segments } S \text{ of } R.
Start(X) \implies \phi \implies \phi \implies \phi [S]_X \phi

Q |- Honest(X) \supset \phi

This is a finitary rule:
• Typical protocol has 2-3 roles
• Typical role has 1-3 receives
• Only need to consider A waiting to receive
Honesty rule  

(Example use)

\( \forall \text{roles R of Q.}\)
\( \forall \text{protocol segments S of R.}\)

\[
\text{Start}(X) \; [ \; ]_X \phi \quad \phi \; [ \; S \; ]_X \phi \\
\hline
Q \vdash \text{Honest}(X) \supset \phi
\]

How this can be used

• If Y receives a message m from X, and
  \( \text{Honest}(X) \supset (\text{Sent}(X,m) \supset \text{Received}(X,m')) \)

• Then Y can conclude
  \( \text{Honest}(X) \supset \text{Received}(X,m') \)

Principal Y can draw conclusions about another principal, X.
Example: Honesty Rule for CR

\[ \text{InitCR}(A, B) = [ \text{new } m; \text{send } A, B, \langle m, A \rangle; \text{receive } B, A, \langle n, \text{sig}_B \{"r", m, n, A \} \rangle; \text{send } A, B, \text{sig}_A \{"i", m, n, B \}];_A \]

\[ \text{RespCR}(B) = [ \text{receive } A, B, \langle m, A \rangle; \text{new } n; \text{send } B, A, \langle n, \text{sig}_B \{"r", m, n, A \} \rangle; \text{receive } A, B, \text{sig}_A \{"i", m, n, B \}];_B \]

For segment 2:

\( \text{Sent}(X,m3) \supset \text{Received}(X,m2) \)

\[ [\text{receive } X, A, \langle x, \text{sig}_X \{"r", m, x, A \} \rangle];_X \]

\( \text{Received}(X,m2) \)

\( \text{Sent}(X,m3) \supset \text{Received}(X,m2) \)

\[ [\text{receive } X, A, \langle x, \text{sig}_X \{"r", m, x, A \} \rangle; \text{send } A, X, \text{sig}_A \{"i", m, x, X \}];_X \]

\( \text{Received}(X,m2) \)

\( \text{Received}(X,m2) \supset (\text{Sent}(X,m3) \supset \text{Received}(X,m2)) \)

For other segments, prove \( \neg (\text{Sent}(X,m3)) \) and derive \( (\text{Sent}(X,m3) \supset \text{Received}(X,m2)) \)
Example complete PCL proof for InitCR

| AM1 | (A.B η) ![A,η] Has(A, A, η) ∧ Has(A, B, η) |
| AN3 | ![A,η] Fresh(A, m, η) |
| AA1 | ![A, η] Send(A, {A, B, m}, η) |
| AA1 | ![A, η] Receive(A, {B, A, n, {m, n, A}}, η) |
| AA1 | ![A, η] Send(A, {A, B, m}, η) |
| AA1 | ![A, η] Verify(A, {m, n, A}, η) |
| AA1 | ![A, η] Send(A, {A, B, m}, η) |
| AA1 | ![A, η] Receive(A, {B, A, n, {m, n, A}}, η) |
| AF1, AF2 | ![A, B, m] (x/B, A, n, {m, n, A}, η) |
| N1 | New(A, m, η) ⊃ ¬New(B, m, η') |
| 5, VER | Honest(B) ∧ Verify(A, {m, n, A}, η) ⊃ |
| | ∃η'.∃m'.((⊥CSend(B, m', η') ∧ (m, n, A) ⊆ m')) |
| HON | Honest(B) ⊃ (∃η'.∃m'.(⊥CSend(B, m', η') ∧ |
| | (m', n, A) ⊆ m' ∧ ¬New(B, m, η')) |
| | ActionsInOrder(Receive(B, A, {m, n, A}, η'), New(B, m, η'), |
| | Send(B, A, {m, n, A}, η')))) |
| 2,3,11, AF3 | Honest(B) ⊃ After(Send(A, {A, B, m}, η), |
| | Receive(B, {A, B, m}, η')) |
| 11, AF2 | Honest(B) ⊃ After(Receive(B, {A, B, m}, η'), |
| | Send(B, A, {n, {m, n, A}}, η')) |
| 11, 4, AF3 | Honest(B) ⊃ After(Send(B, A, {m, n, A}, η'), |
| | Receive(A, {B, A, n, {m, n, A}}, η')) |
| 10 − 13, AF2 | Honest(B) ⊃ η'.(ActionsInOrder(Send(A, {A, B, m}, η), |
| | Receive(B, {A, B, m}, η'), Send(B, B, A, {n, {m, n, A}}, η'), |
| | Receive(A, {B, A, n, {m, n, A}}, η')) |

Table 8. Deductions of A executing Init role of CR
We have a PCL proof. So what?

• Soundness Theorem:
  – If $Q \vdash \phi$ then $Q \models \phi$
  – If $\phi$ is a theorem of PCL then $\phi$ is a valid formula

• Valid: $\phi$ holds in any step in any run of protocol $Q$
  – Unbounded number of participants
  – Dolev-Yao intruder
  – Possibly also for computational model (CPCL)
Using PCL for simple protocols: summary

• Model the protocol
  – Program for each protocol role

• Express security properties
  – Using PCL syntax
  – Authentication, secrecy easily expressed

• Prove security properties
  – Using PCL proof system
    • Using sound implications of pre-conditions and post-conditions
  – Soundness theorem guarantees that provable properties hold in all protocol runs
Protocol composition

• Sequential composition of protocols
  – Run key-exchange protocol
  – Then protocol that uses keys

• Parallel composition
  – Run two protocols in parallel
    • $Q_1 \mid Q_2$ : union of the sets of roles of $Q_1$ and $Q_2$
  – Examples:
    • Many protocols run in parallel, e.g., SSL, IKE, Kerberos
    • In 802.11i, TLS, 4WAY, GroupKey can be run in parallel
Sequential Composition

• Composition rule

\[ \varphi[S]_P \psi \quad \psi[T]_P \theta \]

\[ \varphi[ST]_P \theta \]

• What else do we need?
  – This rule lets us combine local reasoning about sequences of actions
  – But Honesty Rule (invariants) depend on entire protocol
  – How can we combine proofs of invariants?

Same problems for parallel composition
Example: ISO-9798-3

- Shared secret: $g^{ab}$
- Authentication
  - Similar to challenge-response
  - Do we need to prove property from scratch?
• Shared secret: $g^{ab}$
• Authentication
  – Similar to challenge-response
  – Do we need to prove property from scratch?
Sequential Composition

DH

X, Y

new x

X, Y, g^x, x

CR-Init

W, Z, w, x

send W, Z, w, A;
receive Z, W, z, sig_Y{w, z, W};
send W, Z, sig_X{w, z, Z};

X, Y, z^x

ISO

X, Y

new x;
send X, Y, g^x, A;
receive Y, X, z, sig_Y{g^x, z, X};
send X, Y, sig_X{g^x, z, Y};

X, Y, z^x

Sequential composition of roles with term substitution
Abstract challenge response

\[
\begin{align*}
\text{InitACR}(A, X, m) &= [ \\
&\quad \text{send } A, X, \{m\}; \\
&\quad \text{receive } X, A, \langle x, \text{sig}_X\{m, x\}\rangle; \\
&\quad \text{send } A, X, \text{sig}_A\{m, x\}]; \\
\]_A
\end{align*}
\]

\[
\begin{align*}
\text{RespACR}(B, n) &= [ \\
&\quad \text{receive } Y, B, \{y\}; \\
&\quad \text{send } B, Y, \langle n, \text{sig}_B\{y, n\}\rangle; \\
&\quad \text{receive } Y, B, \text{sig}_Y\{y, n\}]; \\
\]_B
\end{align*}
\]

• Role parameters \( m \) and \( n \) instead of nonces

• Specification by modal form: \( \varphi \) [ actions ] \( \psi \)
  – precondition: \( \text{Fresh}(A, m) \)
  – actions: \( [ \text{InitACR} ]_A \)
  – postcondition: \( \text{Honest}(B) \supset \text{Authentication} \)

• Secrecy is proved from properties of Diffie-Hellman
Diffie-Hellman: Property

- Formula
  - true \[ \text{new } a \]_A Fresh(A, g^a)

- Diffie-Hellman property:
  - Can compute $g^{ab}$ given $g^a$ and $b$ or $g^b$ and $a$
  - Cannot compute $g^{ab}$ given $g^a$ and $g^b$
Composition: DH+CR = ISO-9798-3

• Additive Combination
  – DH post-condition matches CR precondition
  – Sequential Composition:
    • Substitute $g^a$ for $m$ in CR to obtain ISO.
    • Apply composition rule
    • ISO initiator role inherits CR authentication.
  – DH secrecy is also preserved
    • Proved using another application of composition rule.

• Nondestructive Combination
  – DH and CR satisfy each other’s invariants
Parallel Composition Theorem (1)

• Honesty rule:

\[ \forall \text{roles } R \text{ of } Q. \]
\[ \forall \text{protocol steps } A \text{ of } R. \]
\[ \text{Start}(X) [ \ ]_X \phi \quad \phi [ A ]_X \phi \]
\[ Q \vdash \text{Honest}(X) \supset \phi \]

• Lemma:

Let \( Q = Q_1 | Q_2 \). If \( Q_1 \vdash \phi \) by proof ending in single use of honesty rule and \( Q_2 \vdash \phi \) similarly, then \( Q \vdash \phi \)

• Proof idea:

Roles (Q) = Roles (Q1) \( \cup \) Roles(Q2)
Parallel Composition Theorem (2)

- **Theorem:**
  
  Let $Q = Q_1 \parallel Q_2$. If $Q_1 \vdash \Gamma$, $\Gamma \vdash \Psi$ and $Q_2 \vdash \Gamma$, then $Q \vdash \Psi$, where $\Gamma$ includes all invariants proved using Honesty rule

- **Proof idea:**
  - By Lemma, $Q \vdash \Gamma$
  - Also, $\Gamma \vdash \Psi$
  - Intuitively, the old proof tree for $Q_1$ still works
General composition pattern

\[ \Gamma \]

\[ \text{DH} \triangleright \text{Honest}(X) \supset \ldots \]

\[ \Gamma \vdash \text{Secrecy} \]

\[ \Gamma \cup \Gamma' \vdash \text{Secrecy} \]

\[ \Gamma \cup \Gamma' \vdash \text{Secrecy} \land \text{Authentication} \ [\text{additive}] \]

\[ \text{DH} \bullet \text{CR} \triangleright \Gamma \cup \Gamma' \ [\text{nondestructive}] \]

\[ \| \]

\[ \text{ISO} \triangleright \text{Secrecy} \land \text{Authentication} \]
Another composition pattern: Protocol Template

Challenge-Response Template

\[
\begin{align*}
A &\rightarrow B: m \\
B &\rightarrow A: n, F(B,A,n,m) \\
A &\rightarrow B: G(A,B,n,m)
\end{align*}
\]

Abstraction

ISO-9798-2

SKID3

ISO-9798-3

Instantiation

\[
\begin{align*}
A &\rightarrow B: m \\
B &\rightarrow A: n, E_{KAB}(n,m,B) \\
A &\rightarrow B: E_{KAB}(n,m)
\end{align*}
\]

\[
\begin{align*}
A &\rightarrow B: m \\
B &\rightarrow A: n, H_{KAB}(n,m,B) \\
A &\rightarrow B: H_{KAB}(n,m,A)
\end{align*}
\]

\[
\begin{align*}
A &\rightarrow B: m \\
B &\rightarrow A: n, \text{sig}_B(n,m,A) \\
A &\rightarrow B: \text{sig}_A(n,m,B)
\end{align*}
\]
STS family

- $STS_0$
  - distribute certificates
  - $m=g^x, n=g^r$
  - $k=g^{xy}$
- $STS_a$
- $STS$
- $STS_P$
- $STS_{PH}$
- $STS_{0H}$
  - open responder
- $STS_{aH}$
- $STS_H$
- $JFK_0$
- $JFK_1$
- $JFK$
- $RFK$

cookie

protect identities

symmetric hash
Sample PCL studies

• Wireless 802.11i
  – Model checking to find errors, improve
  – PCL proof of correctness, including TLS

• Kerberos
  – Including variants “PK-Init” and “DH-init”

• Extensible Authentication Protocol (EAP)
  – Model check to find errors, improve
  – PCL proof of correctness, identify subtleties

• Mesh Security Architecture (IEEE 802.11s)
  – *Motorola group* added some axioms, found problems, identified invariants, proved correctness
802.11i Wireless Authentication

- Supplicant
- UnAuth/UnAssoc
- 802.1X Blocked
- No Key
- 802.11 Association
- Supplicant
- Auth/Assoc
- 802.1X UnBlocked
- PTK/GTK

Diagram:
- 802.11 Association
- EAP/802.1X/RADIUS Authentication
- 4-Way Handshake
- Group Key Handshake
- Data Communication
- MSK
(a) Original Failure Recovery
(b) Improved Failure Recovery
Protocol Composition Logic: PCL

• Intuition
• Formalism
  – Protocol programming language
  – Protocol logic
    • Syntax
    • Semantics
  – Proof System
• Example
  – Signature-based challenge-response
• Composition
• Computational Soundness