

# Computationally Sound Symbolic Analysis of anonymity in presence of active adversaries

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# Summary

- We prove the computational soundness:

$$P \sim Q \quad \rightarrow \quad \llbracket P \rrbracket \approx \llbracket Q \rrbracket$$

Observational equivalence      Computational indistinguishability

- For protocols with PKE and ring signatures, e.g. anonymous voting protocols,

$$P(\text{voter}_1, \text{vote}_1, \text{voter}_2, \text{vote}_2) \sim P(\text{voter}_2, \text{vote}_1, \text{voter}_1, \text{vote}_2)$$

- Without assuming polynomially-computable parsing functions.

# Previous soundness results

- Abadi and Rogaway: term equivalence, **passive** adversary,
- Micciancio and Warinschi: **trace properties**, active adversary, **assume parsing**
- Comon-Lundh and Cortier: process equivalence, **simple processes**, **symmetric encryption**, active adversary, **assume parsing**
- Kawamoto et al.: process equivalence, ring signatures, active adversary, **assume parsing**, **bounded # of sessions**
- Backes et al.: **trace properties**, active adversary, **restricted class of protocols?**, **assume parsing**

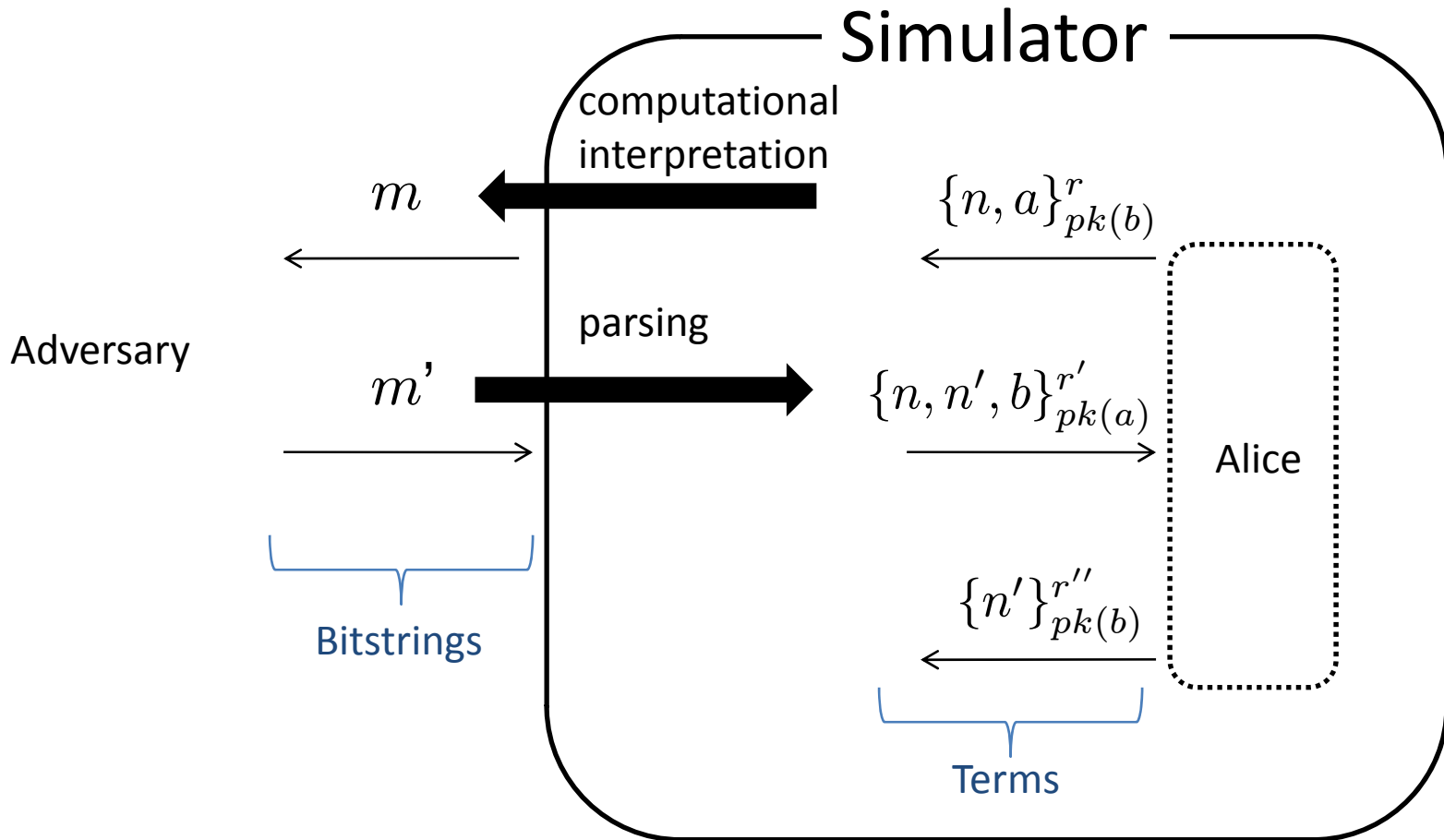
# Our result

- We prove the computational soundness:
  - Process equivalence
  - Larger class of protocols
  - More primitives (PKE and ring signatures)
  - Active adversary
  - Unbounded number of sessions
  - *Without assuming parsing*

# Simulator's parsing

- In the proof of mapping lemma [Micciancio and Warinschi] , the simulator simulates agents by interpreting terms and parsing bitstrings.
- If a message sent by the adversary is parsed into a term symbolically impossible to be produced, the simulator break, e.g., IND-CCA.
- Parsing must be polynomially computable.

# Simulator's parsing

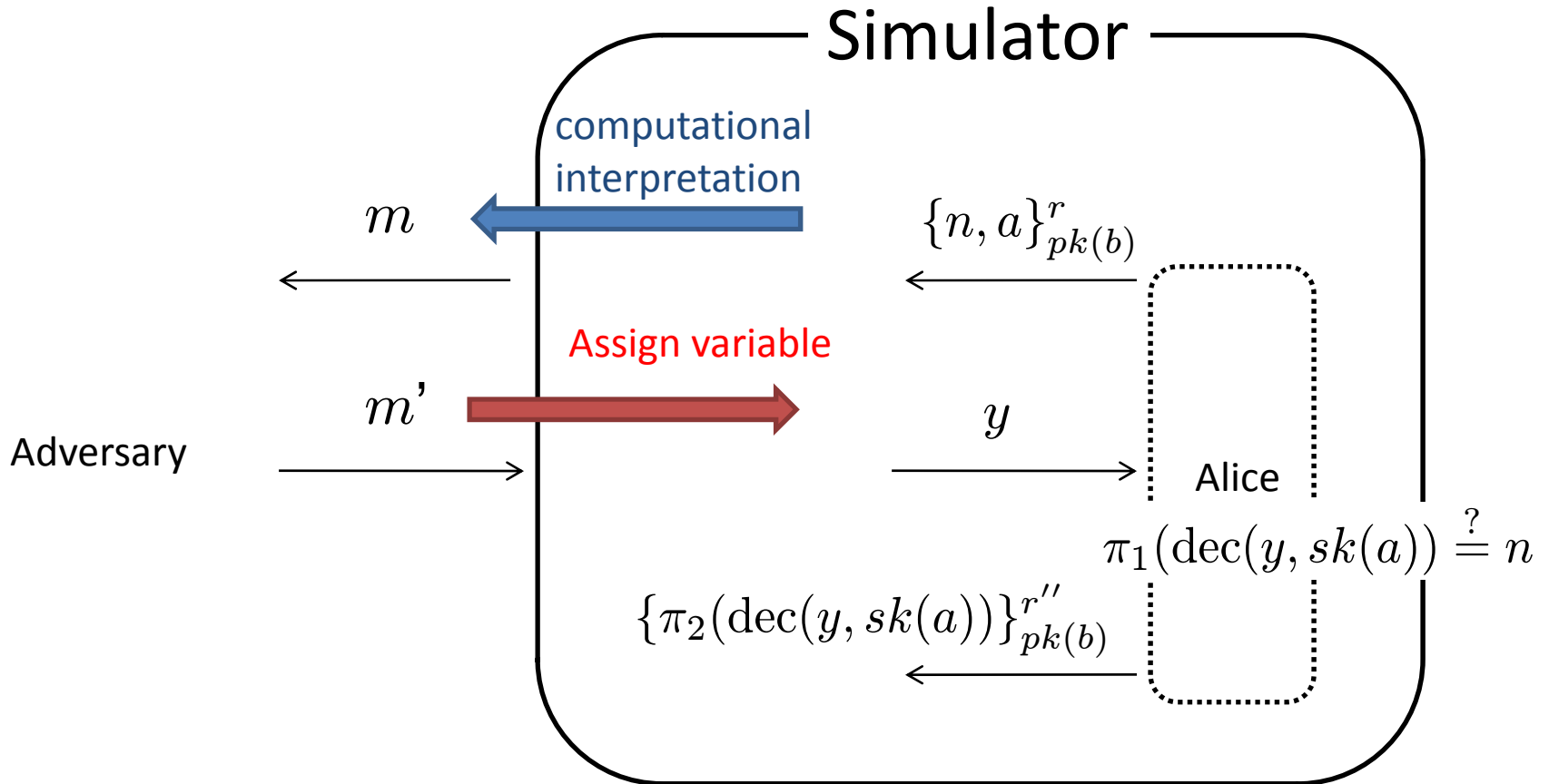


In order to make parsing polynomially computable,  
Previous work doesn't allow a ciphertext by an unknown key.

# Our Approach

- Simulator doesn't parse messages, but only simulates participants.  
(It doesn't touch messages until participants do)
- If the simulator goes down to a branch (of *if-then-else* statement) that is symbolically impossible, we break, e.g., IND-CCA.

# Our approach



The simulator does not touch received messages until the participants do in a condition or a reply message.



# Processes (Applied pi-calculus)

## Processes

$$\begin{aligned} P, Q ::= & P \parallel Q \\ & !P \\ & \nu n.P \\ & c(x).P \quad \text{Receiving of term} \\ & \bar{c}(M).P \quad \text{Sending of term M} \\ & \text{if } C \text{ then } P \text{ else } Q \end{aligned}$$

## Terms

$$\begin{aligned} M ::= & n, r \\ & \{M, M'\} \\ & \pi_i(M) \\ & \{M\}_{M'}^r \\ & \text{dec}(M, M') \\ & [M]_{M',VK}^r \\ & \text{check}(M, VK) \end{aligned}$$

## Communication

$$c(x).P \parallel \bar{c}(M).Q \rightarrow P\{x \mapsto M\} \parallel Q$$

Observational equivalence (P and Q behaves same under any process A)

$$P \sim Q \iff \forall A. ((P \parallel A) \downarrow_c \Leftrightarrow (Q \parallel A) \downarrow_c)$$

# Outline of the soundness proof

[Comon-Lundh and Cortier]

$$P \sim Q$$



Observational equivalence is characterized by equivalence (bisimilarity) of computation tree

$$T_P \sim T_Q$$



Generalization of [Abadi and Rogaway] into trees

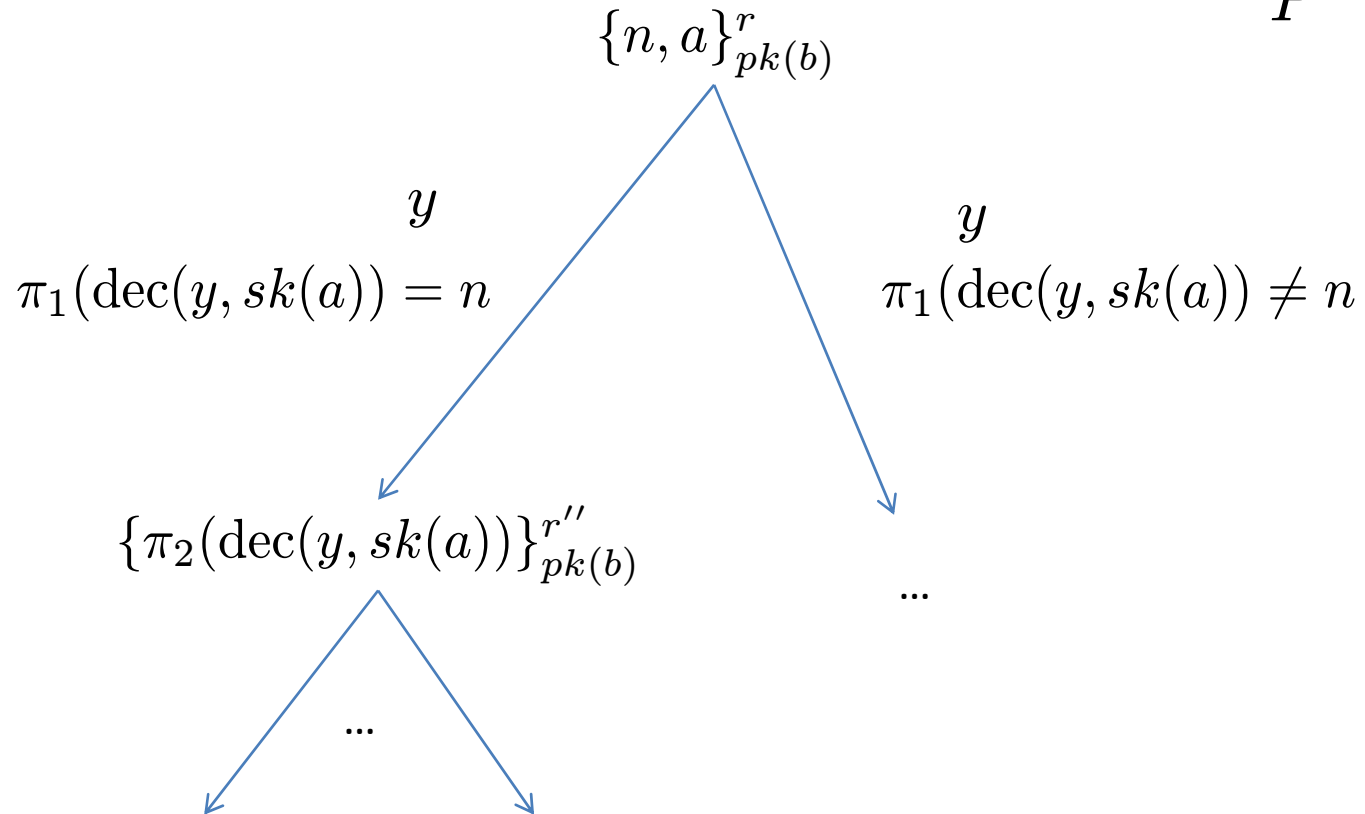
$$\mathcal{O}_{T_P} \approx \mathcal{O}_{T_Q}$$



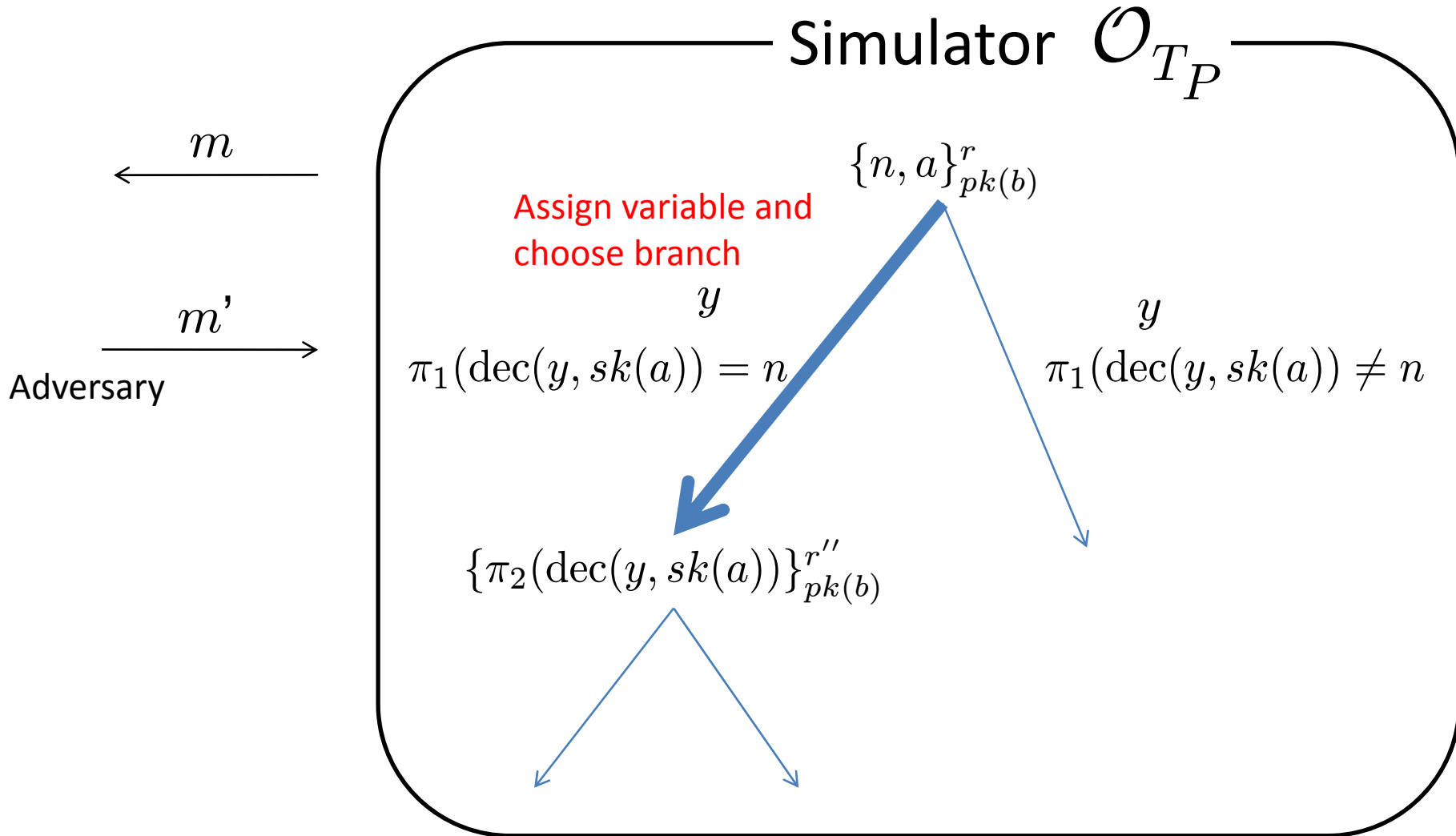
Mapping Lemma  
[Micciancio and Warinschi]

$$\llbracket P \rrbracket \approx \llbracket Q \rrbracket$$

# Our Computation Tree

 $T_P$ 

# Simulator



# More details

Successive game transformations  
(IND-CCA, anonymity of ring signatures)

$$\mathcal{O}_{T_P} \approx \mathcal{O}_{\Omega_{pk(a)}(T_P)} \approx \mathcal{O}_{\Omega_{pk(b)}(\Omega_{pk(a)}(T_P))} = \mathcal{O}_{\Omega_{pk(b)}(\Omega_{pk(a)}(T_Q))} \approx \mathcal{O}_{\Omega_{pk(a)}(T_Q)} \approx \mathcal{O}_{T_Q}$$

$\approx$   $\approx$   $\approx$   $\approx$   $\approx$   $\approx$

$\llbracket P \rrbracket$   $\llbracket Q \rrbracket$

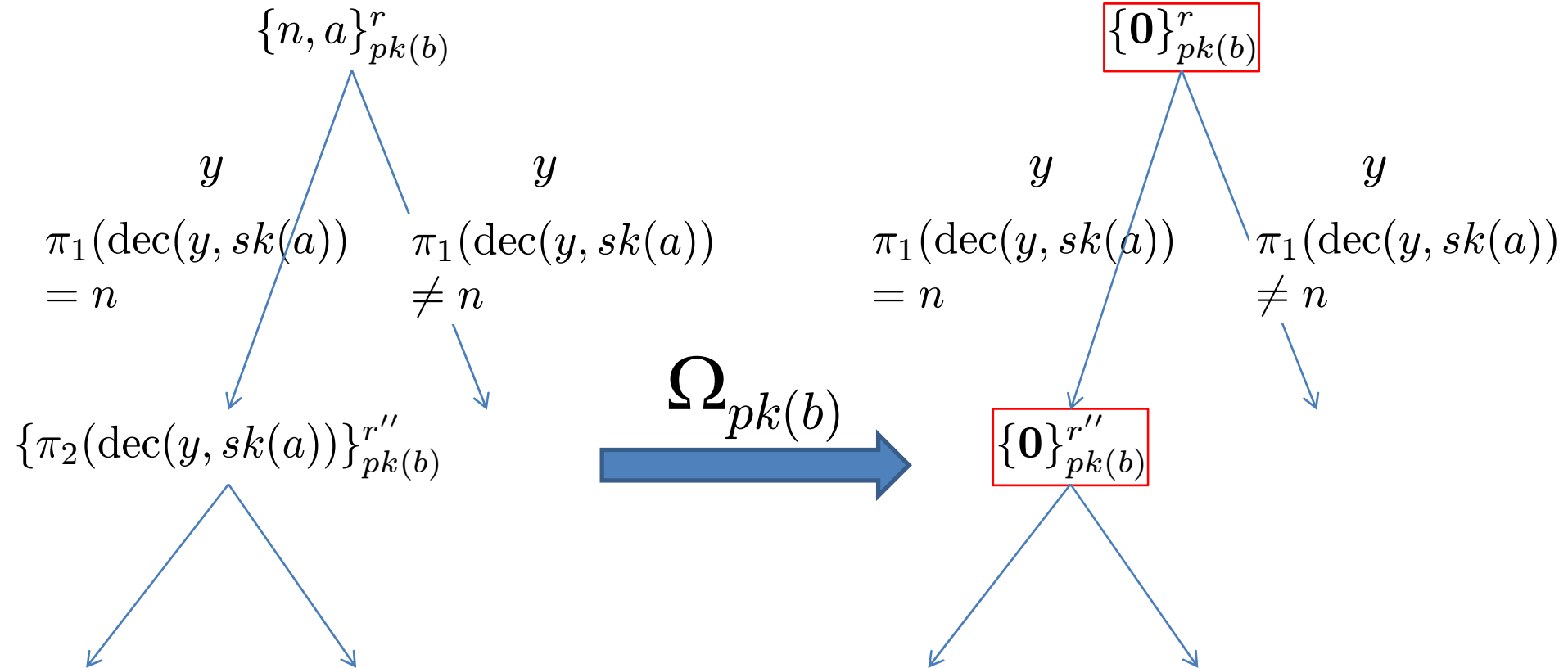
Mapping Lemma  
(Unforgeability, IND-CCA)

# Computational assumptions

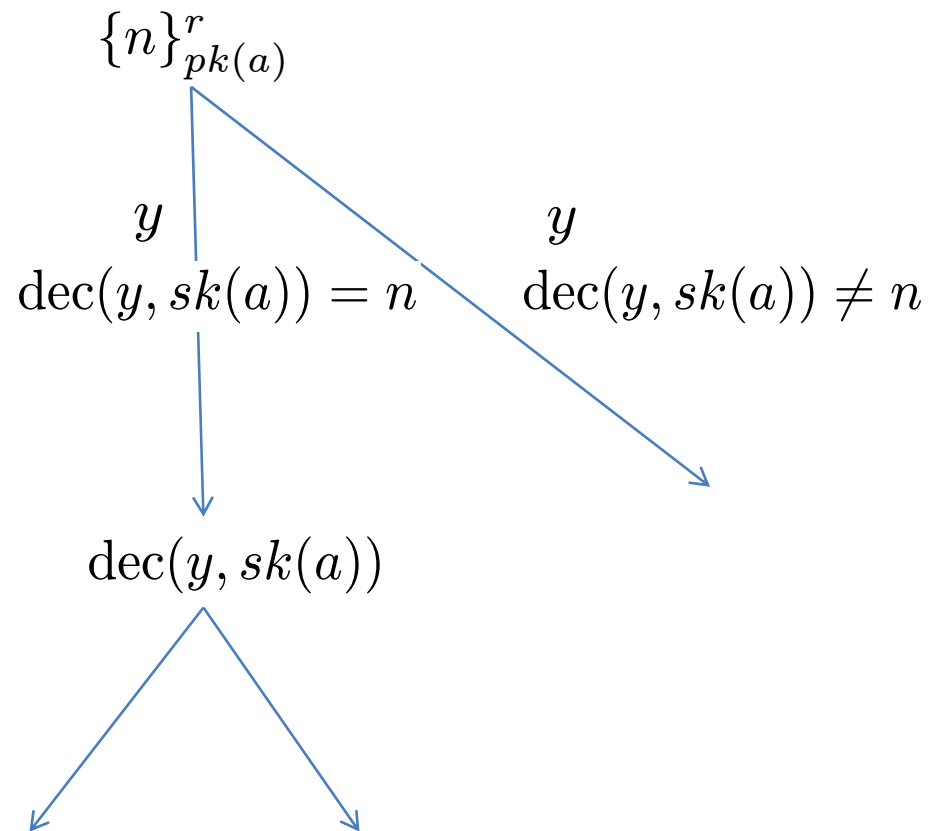
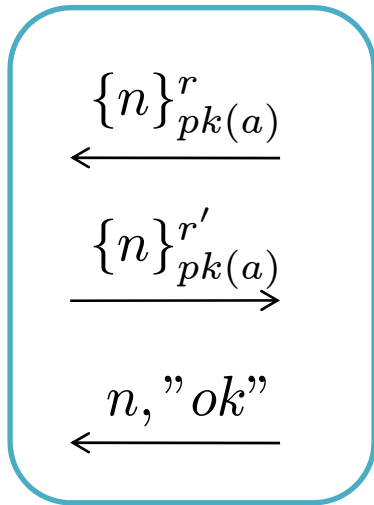
- Public-key encryption: IND-CCA
- Ring signatures:
  - Unforgeability: signatures cannot be produced without signing keys
  - Anonymity: signers of a group are indistinguishable from their signatures

$$[m]_{sk_0, \{vk_0, vk_1, vk_2\}}^r \approx [m]_{sk_1, \{vk_0, vk_1, vk_2\}}^r$$

# Transformation by IND-CCA: Easy case

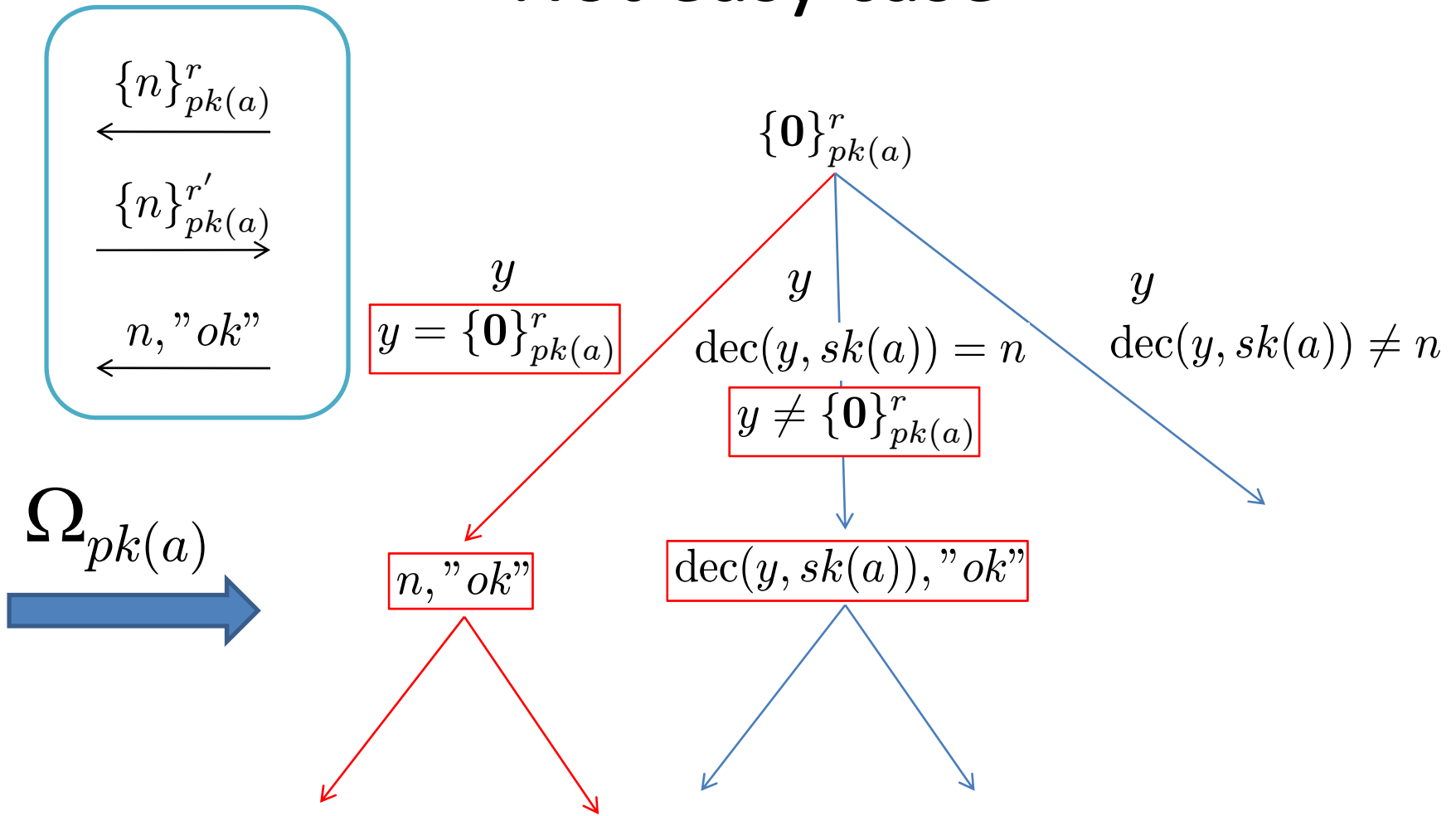


# Transformation by IND-CCA: Not easy case





# Transformation by IND-CCA: Not easy case



# Conclusion

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